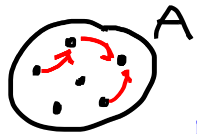
R, S relace na A



$R = \{(a,b), (e,y), \dots\}$

~~$a \in R$~~ $(a,b) \in R$ $a, b, c, d, \dots \in A$ aRb

---

1 R, S symetrické $\Longrightarrow$ $R \cup S$ symetrické T

Dk: P R, S symetr.

$a, b \in A$: $\underbrace{(a,b) \in R \cup S}_{}$ $\longrightarrow$ $\left[\begin{array}{l} (a,b) \in R \\ \text{nebo} \quad (a,b) \in S \end{array}\right]$

$a\,(R \cup S)\,b$

$(a,b) \in R \xrightarrow[R]{P} (b,a) \in R$

nebo

$(a,b) \in S \xrightarrow[S]{P} (b,a) \in S$

$\Longrightarrow \underbrace{(b,a) \in R \cup S}_{b\,(R \cup S)\,a}$

② R, S tranzitivní $\Longrightarrow$ $R \cup S$ tranz. F

Dk: Ⓟ R, S tranz.

$a, b, c \in A$: $(a, b) \in R \cup S \wedge (b, c) \in R \cup S \longrightarrow$

$[(a,b) \in R \vee (a,b) \in S] \wedge [(b,c) \in R \vee (b,c) \in S] \longrightarrow$

R R $\rightarrow$ $(a,c) \in R \rightarrow \in R \cup S$

S S $\rightarrow$ $(a,c) \in S \rightarrow \in R \cup S$

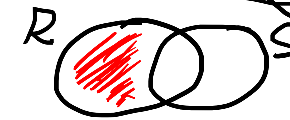
S R $\rightarrow$

$(a,b) \in R \wedge (b,c) \in S \longrightarrow$? $\longrightarrow (a,c) \in R \cup S$

R

S

③ R, S antisym $\Longrightarrow$ $R - S$ antisym (T)



Dk: (P) R, S antisym

$a, b \in A$ $\quad (a,b) \in R - S \wedge (b,a) \in R - S \longrightarrow$

$\binom{a}{b}, \binom{c}{d}$ $\longrightarrow [(a,b) \in R \wedge (a,b) \notin S] \wedge [(b,a) \in R \wedge (b,a) \notin S]$

$(\binom{a}{b}, \binom{c}{d}) \in T$ $\longrightarrow [(a,b) \in R \wedge (b,a) \in A] \wedge [(a,b) \notin S \wedge (b,a) \notin S]$

$ax+b$
$cx+d$

(P)

$\xrightarrow{R} \underline{\underline{a = b}}$

$\binom{a}{b} = \binom{c}{d}$

$a = c$
$b = d$

$(ax+b, cx+d) \in T$

$ax+b = cx+d$

$\alpha \wedge \beta \wedge \gamma$ $\quad \{\alpha, \beta, \gamma\}$ $\quad \{\alpha, \beta, \gamma, \delta, \varepsilon\}$ $\longrightarrow$ (Z)

4 R reflex $\Longrightarrow$ $R-S$ reflex

$\langle S$ antireflex$\rangle$

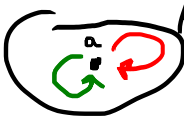
F

Dk: P R reflex

T

$\underline{a \in A}$: $\longrightarrow$ bonus: libov. $a \in A$ → R reflex $\rightarrow (a,a) \in R$; $\wedge$ S antireflex $\rightarrow (a,a) \in S$

$$\left[\underbrace{(a,a) \in R}_{P} \wedge (a,a) \notin S\right] \longrightarrow \underline{(a,a) \in R-S}$$



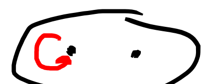
A

a

R

S

$R - S = \emptyset$



$A = \{a\}$

$R = \{(a,a)\}$ $S = \{(a,a)\}$

$R, S \text{ sym} \Longrightarrow \overset{R\cap S}{R \cup S} \text{ sym}$

P

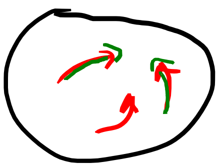
$a, b \in A \,,\; (a,b) \in \overset{R\cap S}{R \cup S} \rightarrow \left\{ \begin{array}{l} (a,b) \in R \\ \wedge\, (a,b) \in S \end{array} \right\} \xrightarrow{P}$

$\rightarrow \left\{ \begin{array}{l} (b,a) \in R \\ \wedge\, (b,a) \in S \end{array} \right\} \rightarrow\; \rightarrow (b,a) \in \overset{R\cap S}{R \cup S}$

$$\underbrace{R \text{ sym}, \ R \subseteq S \implies \underline{\underline{S \text{ sym}}}}$$

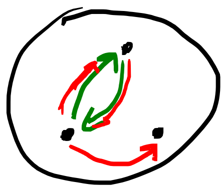
Dk: (P) $\underset{\sim}{R \text{ sym}}$, $R \subseteq S$

$a, b \in A$ $\quad \underline{\underline{(a,b) \in S}} \longrightarrow$



$R \subseteq S$

p - p



$\longrightarrow \underline{(b,a) \in S}$

$R = \{(a,b), (b,a)\}$

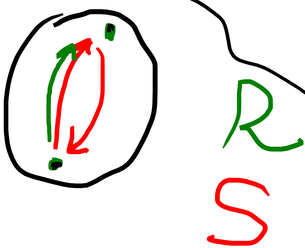
$S = \{ -,- \ , (b,c)\}$

S sym, $R \subseteq S \implies R$ sym $\quad$ F

P1 S sym, P2 $R \subseteq S$

$a, b \in A$: $\underline{(a,b) \in R} \xrightarrow{P_2} (a,b) \in S \xrightarrow{P_1}$

$\to (b,a) \in S \to \underline{(b,a) \in R}$



R, S na A sym $\Longrightarrow$ $S \circ R$ sym

Dk: (P) R, S sym

$a, b \in A$

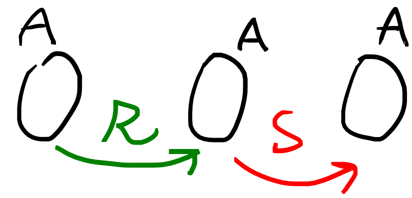
$(a,b) \in S \circ R \rightarrow$

$\xrightarrow{(P)}$ $\exists x \in A: aRx \wedge xSb$

$\rightarrow xRa \wedge bSx \rightarrow bSx \wedge xRa$

$(b,a) \in R \circ S$

$\rightarrow (b,a) \in S \circ R$



$a \xrightarrow{R} x \xrightarrow{S}$

$bRy \wedge ySa$

$\downarrow$

$(b,a) \in S \circ R$

$$R \text{ sym} \Longrightarrow R \circ R \text{ sym} \quad R^2$$

Dk P R sym

$a, b \in A$, $\underline{a (R \circ R) b} \longrightarrow \exists x \in A : aRx \wedge xRb$

$\xrightarrow{P} xRa \wedge bRx \longrightarrow \underline{bRx} \wedge \underline{xRa}$

$\longrightarrow \underline{b (R \circ R) a}$

$(a,x) \in R$	$(x,b) \in R$
$(x,a) \in R$	$(b,x) \in R$
$(b,x) \in R$	$(x,a) \in R$

//