$y' = \frac{3y}{x-1}$ $\longrightarrow$ obecné řeš

$\longrightarrow$ part. řeš. a) $y(-1) = -8$; b) $y(2) = 8$; c) $y(1) = 8$.

Omezení: $x \neq 1$.

Obec. řeš: separ: ✓ $y' = \underbrace{\frac{1}{x-1}} \cdot \underbrace{3y}$

$\frac{dy}{dx} = \frac{3y}{x-1}$ $\xrightarrow{y=0}$ Stac. řeš.: 1 $y(x) = 0,\ x \in (-\infty, 1)$.

$y(x) = 0,\ x \in (1, \infty)$.

$y \neq 0$:

$\int \frac{dy}{y} = \int \frac{3 \cdot dx}{x-1}$ $\{h = x-1\}$

$\ln|y| = 3 \cdot \ln|x-1| + C$

$e^{\ln|y|} = \underline{e^{3 \cdot \ln|x-1| + C}}$

$\longrightarrow |y| = e^{C} \cdot e^{3 \cdot \ln|x-1|}$

$y = \pm e^{C} \cdot |x-1|^3$

$y(x) = D \cdot (x-1)^3$ 2 $D \neq 0$

$$\boxed{y(x) = D \cdot (x-1)^3 \quad x \neq 1.} \quad \boxed{\text{obecné řeš.}}$$

Pozn!:

$$y(x) = \pm e^C \cdot |x-1|^3 \qquad |x-1|^4 = (x-1)^4$$

$$= \pm e^C \cdot \left(\left(\pm (x-1) \right)^3 \right)$$

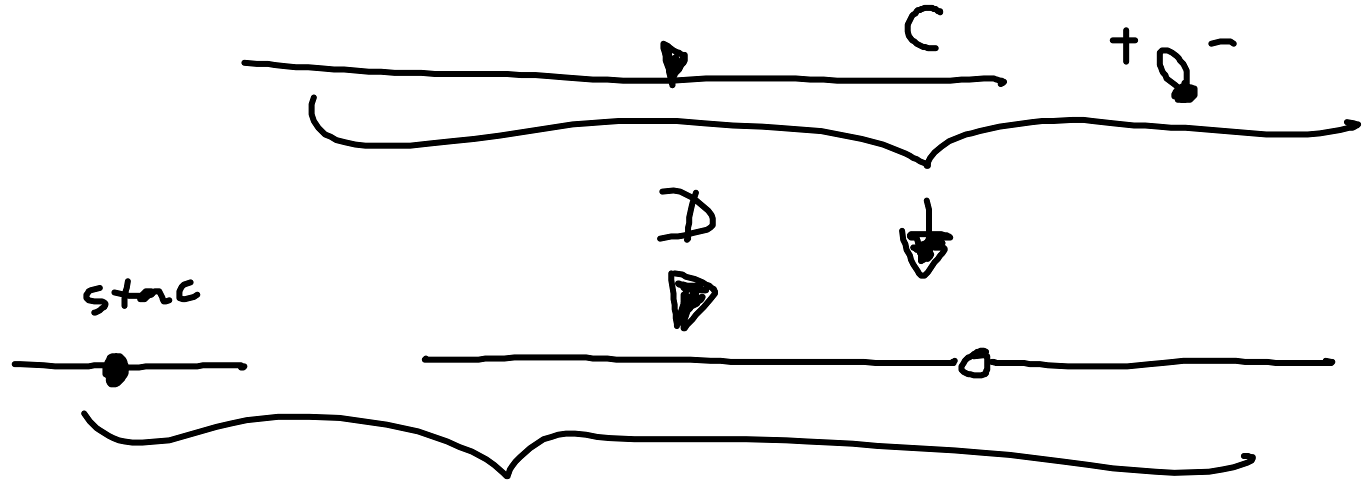
$(+1) \cdot (-1)^3$

$$= \pm e^C \cdot (\pm 1)^3 \cdot (x-1)^3$$

$(\pm 1)^4 = +1$

$$= \pm \cdot (\pm) \cdot e^C \cdot (x-1)^3$$

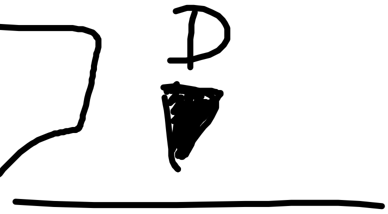
$$= \underline{\pm e^C} \cdot (x-1)^3$$



$y' = y^2$ → $y = 0$ s. r.

$\int \frac{dy}{y^2} =$

$-\frac{1}{y} = \dots \rightarrow y = \dots$ C

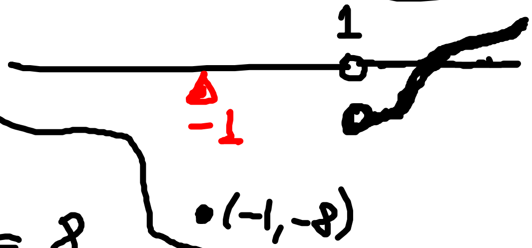


$$y(x) = D\cdot(x-1)^3 \qquad x \neq 1.$$

a) $y(-1) = -8$ chci: $D\cdot((-1)-1)^3 \overset{?}{=} -8$

$D\cdot(-8) \overset{?}{=} -8 \rightarrow \underline{D=1}$

$\boxed{y_a(x) = (x-1)^3,\ x\in(-\infty, 1)}$



b) $y(2) = 8$. chci: $D\cdot(2-1)^3 \overset{?}{=} 8$

$\hookrightarrow D = 8$

$\boxed{y_b(x) = 8\cdot(x-1)^3,\ x\in(1,\infty).}$

c) chci: $D\cdot(1-1)^3 = 8$

$0 = 8$??

$y_c(x)$ neex.

$$\underbrace{y_a(x) = (x-1)^3 = -|x-1|^3.}$$

na $(-\infty, 1) \rightsquigarrow (x-1)^3 < 0$

$$y(x) = D\cdot(x-1)^3 \sim \boxed{Dx^3}.$$

asymptotická rychlost růstu

$$\boxed{y' = \frac{3y}{x-1}}$$

$$y' + a(x) \cdot y = b(x)$$

$$y' - \frac{3y}{x-1} = 0$$

$$y' + \left(\frac{-3}{x-1}\right) \cdot y = 0$$

Lineární

$\boxed{\text{homogenní}}$

$$2y' = \frac{e^x}{y}$$

a) $y(0) = 1$; b) $y(0) = -1$;

c) $y(1) = 0$; d) $y(1) = 1$.

---

lineární?

$$y' - e^x \cdot \frac{1}{y} = 0 \quad / y^2$$

ne!

$a(x) \cdot y$

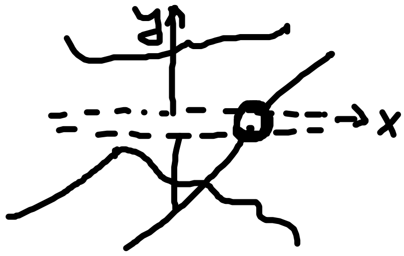
$$\cancel{y^2} \cdot y' - e^x \cdot y = 0$$

separabilní?

$$y' = \underbrace{\tfrac{1}{2}e^x}\cdot \underbrace{\frac{1}{y}}$$

ano

$2y' = \dfrac{e^x}{y}$, omez: $y \neq 0$.



obec. řeš: $2\dfrac{dy}{dx} = \dfrac{e^x}{y}$

$$\int 2y\,dy = \int e^x\,dx$$

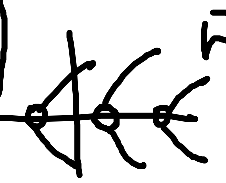
$$y^2 = e^x + C$$

$y \neq 0 \Leftrightarrow \sqrt{e^x + C} \neq 0$ (1)

$\Leftrightarrow e^x + C \neq 0$

$$\boxed{y(x) = \pm\sqrt{\underbrace{e^x + C}_{(2)\ \geq 0}}, \quad \underline{e^x + C > 0}}$$

obecné řeš.



a) $y(0)=1$; b) $y(0)=-1$;

c) $y(1)=0$; d) $y(1)=1$.

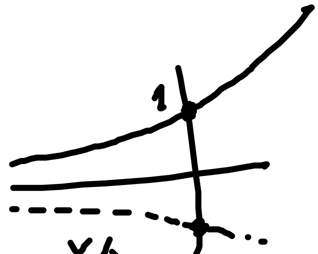
$y(x)=\pm\sqrt{e^x+c}$

$\boxed{e^x+c>0}$

a) chci $\cancel{\pm}\sqrt{e^0+c} \overset{?}{=} 1$ $\quad +\sqrt{1+c}=1$

$c=0$.

$$\boxed{y_a(x)=\sqrt{\underbrace{e^x}_{>0}}=e^{x/2},\ x\in\mathbb{R}}$$



b) chci $\cancel{\pm}\sqrt{e^0+c} \overset{?}{=} -1$

$-\sqrt{1+c}=-1$, $c=0$ $\quad \boxed{y_b(x)=-\sqrt{e^x}=-e^{x/2},\ x\in R}$

c) chci: $\pm\sqrt{e^1+c} \stackrel{?}{=} 0 \iff e+c=0$

$c=-e$

$y(x) \pm \sqrt{e^x-e}$, ??

$y_c(x)$ neex.

$2yy' = e^x$

c) $y(x) = \pm\sqrt{e^x-e}$

$x \in \langle 1, \infty)$

d) chci: $\pm\sqrt{e^1+c} = 1$

$\sqrt{e+c} = 1 \iff$

$\iff e+c=1 \iff c = 1-e$

$\boxed{y_d(x) = \sqrt{e^x+1-e},\ x \in (\ln(e-1), \infty)}$

nutno: $e^x+1-e>0$

$e^x > e-1$

$x > \ln(e-1)$

$$y' = \frac{(e^x + x)\cdot(y-1)}{(x^2+1)(y+x)}$$

$$\frac{dy}{y-1}$$

$y = 1$

$\boxed{y(x) = 1}$ ✓

$t \dots$ čas $\quad x \dots$ pozici $\quad \dot{x} = 2tx^2$

$$\frac{dx}{dt} = 2tx^2$$

$$\cancel{\frac{dy}{dx} = 2xy^2}$$

a) $x(-2) = 1$
b) $x(-1) = -1$
c) $x(1) = -\frac{1}{2}$
d) $x(3) = -\frac{1}{4}$
e) $x(1) = 0$

obecné ř: $\rightarrow$ stac. ř. $x(t) = 0$, $t \in R$

$x \neq 0$

$$\int \frac{dx}{x^2} = \int 2t\, dt$$

$$-\frac{1}{x} = t^2 + C$$

$$-\frac{1}{t^2+C} = x$$

$$\boxed{x(t) = \frac{-1}{t^2+C},\ t^2+C \neq 0}$$

$$\int x^{-2} dx = \frac{x^{-2+1}}{-2+1}$$

obecné řeš:

$$\boxed{x(t) = \begin{cases} \dfrac{-1}{t^2+c}, & t^2+c \neq 0 \\ 0, & t \in \mathbb{R} \end{cases}}$$

a) $x(-2) = 1$

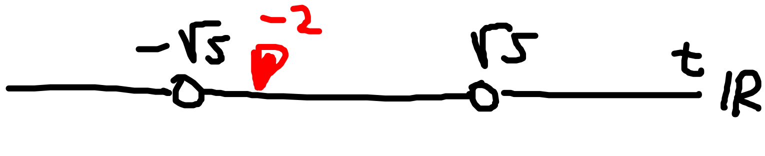
$$\frac{-1}{(-2)^2+c} = 1$$

$$-1 = 4 + c$$

$$-5 = c$$

$$\boxed{x_a(t) = \frac{-1}{t^2-5} = \frac{1}{5-t^2}}, \quad t \in (-\sqrt{5}, \sqrt{5})$$

$t^2 \neq 5 \quad t \neq \pm\sqrt{5}$



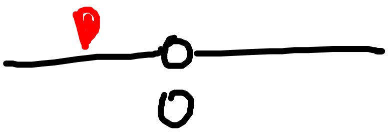
b) $x(-1) = -1$

$$\frac{-1}{(-1)^2+c} = -1$$

$$-1 = -(1+c)$$

$$c = 0$$

$$x_b(t) = \frac{-1}{t^2}, \quad t \in (-\infty, 0)$$



c) $x(1) = -1/2$

$$\frac{-1}{1^2+C} = -1/2$$

$$2 = 1 + c$$

$$c = 1$$

$$x_c(t) = \frac{-1}{t^2+1}, \quad t \in \mathbb{R}$$

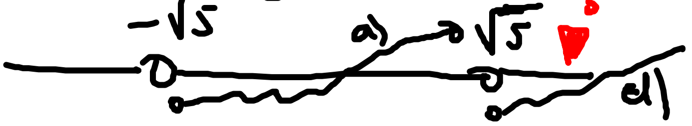
d) $x(3) = -\frac{1}{4}$

$$\frac{-1}{3^2+c} = -\frac{1}{4}$$

$$4 = 9 + c$$

$$-5 = c$$

$$\boxed{x_d(t) = \frac{-1}{t^2-5} = \frac{1}{5-t^2}, \quad t \in (\sqrt{5}, \infty)}$$



e) $x(1) = 0$

$$\frac{-1}{1^2+c} = 0$$

~~$x_e(t)$ neex~~

$$-1 \overset{?}{=} 0$$

$$\boxed{c = ?}$$

$$\boxed{x_e(t) = 0, \quad t \in \mathbb{R}}$$