$y' - \cos(x)\,y = -\cos(x)$ | obecné řeš?

separ?

$$y' = \cos(x)\,y - \cos(x)$$

$$y' = \underbrace{\cos(x)}_{g(x)} \cdot \underbrace{(y-1)}_{h(y)}$$

✓ ano

lineární?

$\boxed{\text{ano!}}$

$$y' + \underbrace{(-\cos(x))}_{a(x)} \cdot y = \underbrace{-\cos(x)}_{b(x)}$$

$\boxed{\text{Variace}}$

1) homogenní: $y' - \cos(x)\, y = 0$

$$\frac{dy}{dx} = \cos(x)\, y$$

$y=0$ stac. ř.

$$\int \frac{dy}{y} = \int \cos(x)\, dx$$

$$\ln|y| = \sin(x) + C$$

$$|y| = e^{\sin(x) + C}$$

$$y = \underbrace{\pm e^{C}}_{D} \cdot e^{\sin(x)}$$

$$y_h(x) = D \cdot e^{\sin(x)}$$

$D = 0 \rightarrow$ stac. ř.

2) variace: $\overset{\text{odhad}}{=} \quad y(x) = \boxed{D(x) \cdot e^{\sin(x)}}$

Dosadím: $\left[D(x)\cdot e^{\sin(x)}\right]' - \cos(x)\cdot\left[D(x)e^{\sin(x)}\right] = -\cos(x)$

$$D'(x)e^{\sin(x)} + \cancel{D(x)\cdot e^{\sin(x)}\cdot\cos(x)} - \cancel{\cos(x)\,D(x)\,e^{\sin(x)}} = -\cos(x)$$

$$D'(x)\,e^{\sin(x)} = -\cos(x) \iff D'(x) = e^{-\sin(x)}\cdot(-\cos(x))$$

$$D(x) = \int \underbrace{e^{-\sin(x)}}_{e^{w}}\ \underbrace{(-\cos(x))\,dx}_{dw} = \left\{\begin{matrix} w = -\sin(x) \\ dw = -\cos(x)\,dx \end{matrix}\right\} = \int e^{w}\,dw$$

$$= e^{w} + C = e^{-\sin(x)} + C$$

Řešení: $y(x) = \left(e^{-\sin(x)} + C\right)\cdot e^{\sin(x)}$ %

obecné řešení:

$$\boxed{y(x) = \left(e^{-\sin(x)} + C\right) \cdot e^{\sin(x)} = 1 + C e^{\sin(x)}, \quad x \in \mathbb{R}}$$

Alt: $D(x) = e^{-\sin(x)} \rightsquigarrow y_p(x) = e^{-s} \cdot e^{s} = 1$

obecné: $\boxed{y = y_p + y_h = 1 + D e^{\sin(x)}, \quad x \in \mathbb{R}}$

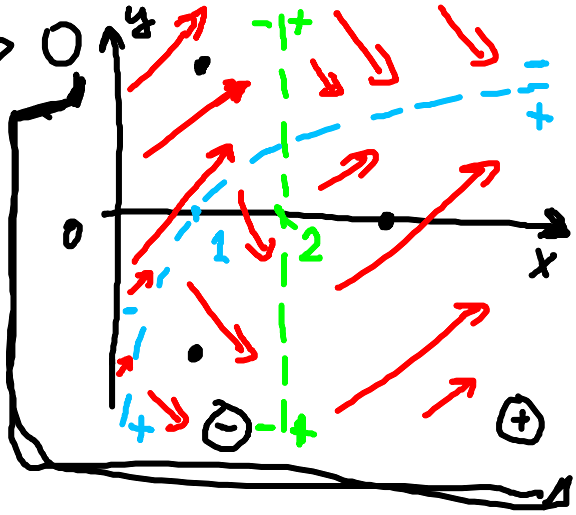
$$y' = (x-2)\cdot(\ln(x) - y) \longrightarrow x > 0$$

ODR

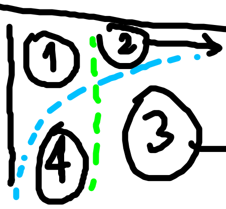
$\longrightarrow y' \overset{?}{=} 0:$ $x = 2$

$\ln(x) - y = 0 \longrightarrow y = \ln(x)$

y' neex: X



$\longrightarrow$



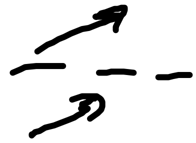
3 $\longrightarrow (3,0) \to y' = 1\cdot(\ln(3) - 0)$ +

$\longrightarrow$ 4 $\longrightarrow (1,-2) \xrightarrow{\text{ODR}} y' = \underbrace{(1-2)}\cdot(\overbrace{\ln(1)}^{0} \underbrace{-(-2)}_{+2})$ −

$y = \ln(x)$ ↑ $y > \ln(x) \longrightarrow (\ln(x) - y) < 0$

↓ $y < \ln(x) \longrightarrow (\ln(x) - y) > 0$

$$\boxed{y' = (x-2)\cdot(\ln(x) - y)}$$



stac. řeš.: $y(x)$ konst. $= y_0$

konst.

$y' = 0$ vždy $(\forall x)$

$$(x-2)\cdot(\ln(x) - y) \overset{?}{=} 0$$

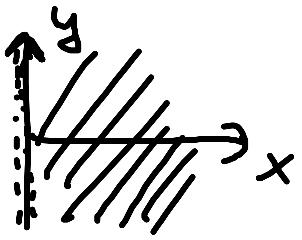
[najdu $y_0 \in \mathbb{R}$ aby $y' = 0$ vždy?]

$y = \ln(x)$

neex. stac. řešení!

$y' = (x-2)\cdot(\ln(x) - y)$

$\boxed{x > 0}$ existence

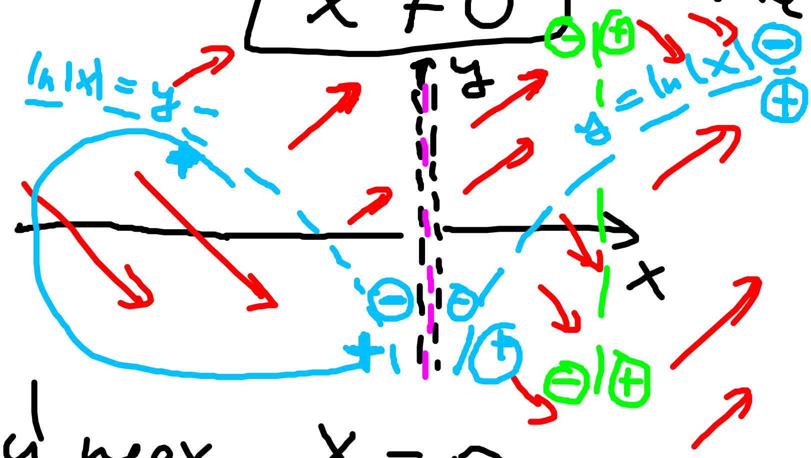


y' neex

$x = 0$

$y' = \underbrace{(x-2)}\cdot\underbrace{(\ln|x| - y)}$

$\boxed{x \neq 0}$ existence



y' neex $\quad x = 0$

$y' = x - y$

+/-

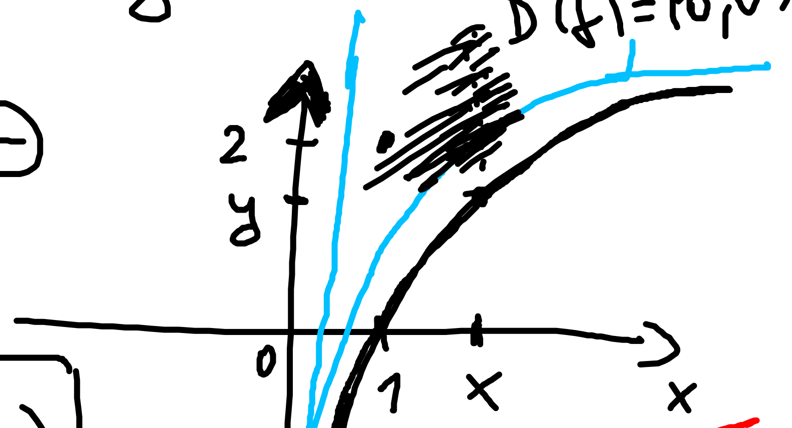
$\boxed{\ln(x) - y} \rightarrow y = \ln(x)$

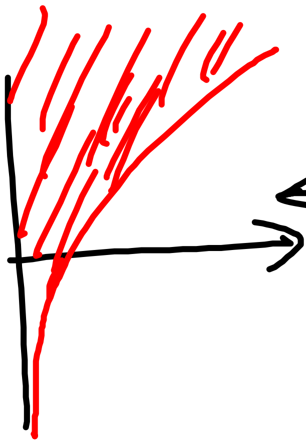
$x = 1$

$y = 2$

$\ln(1) - 2 = -2 \quad -$

$D(f) = (0, \infty)$





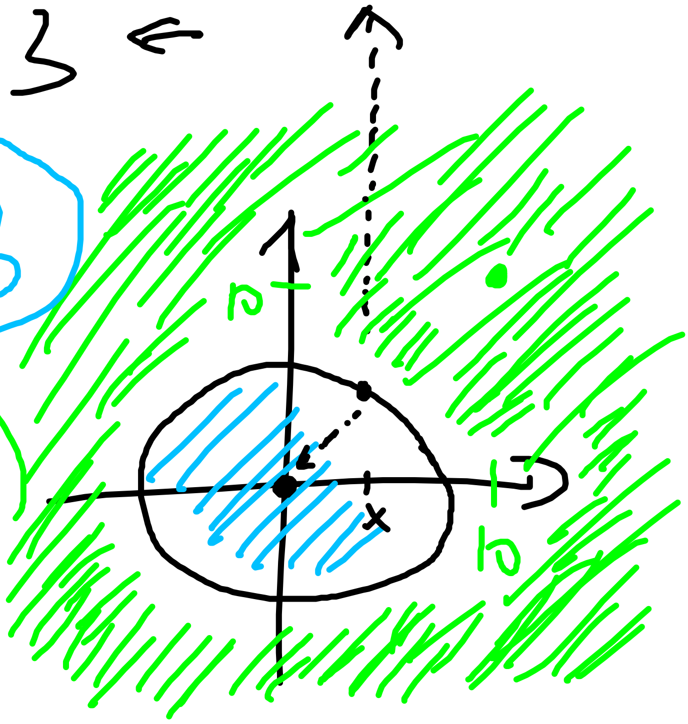
$\boxed{y > \ln(x)}$

$\boxed{y < \ln(x)}$

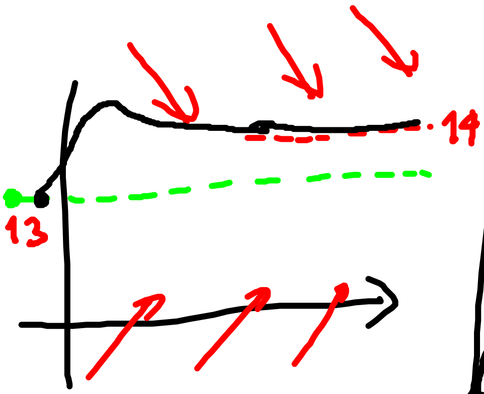
$$x^2 + y^2 = 13$$

$$x^2 + y^2 < 13$$

$$x^2 + y^2 > 13$$



$$y' = \frac{\cancel{x}}{y-13}$$



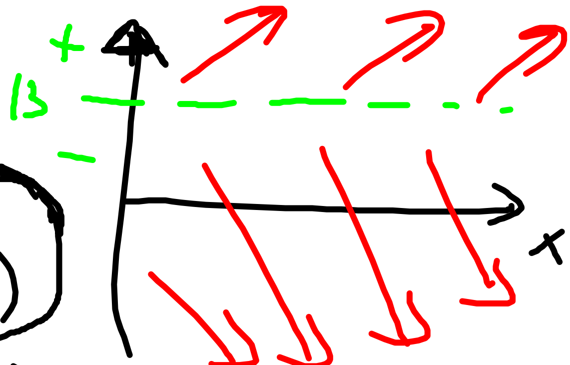
$$y' = \frac{x^{y}}{y-13} \to e^{y\cdot\ln(x)} \quad (+)$$

$$x > 0$$

y' nex

$y = 13$

$$y' = x^{y} \cdot (y-13)$$



$\ll y(x) = 13$ stac. řeš. (nestabilní)

$$y' = \frac{y^2}{1-y}$$ → vekt. pole → stac. řeš. → stabilita

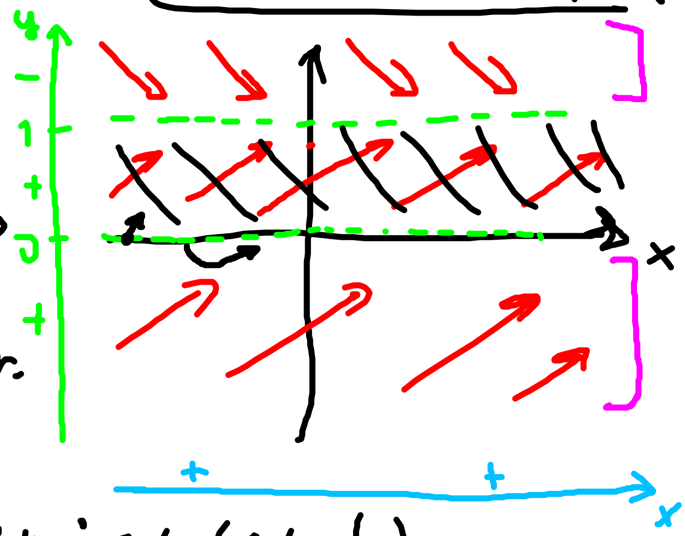
→ exist: $y \neq 1$

→ dělicí: $y = 0$, $y = 1$ $\langle y' = 0 \rangle$ $\langle y'$ neex$\rangle$

→ stac. řeš: $\boxed{y_0 = 0}$ ← ekvilibr.

↳ $\boxed{y(x) = 0,\ x \in \mathbb{R}}$

→ nestabilní (semistabilní)



$y' = y^2(1-y)$ → stabilní stac. řeš: $y(x) = 1,\ x \in \mathbb{R}$

$$y' = y^2 \cdot (1-y)$$

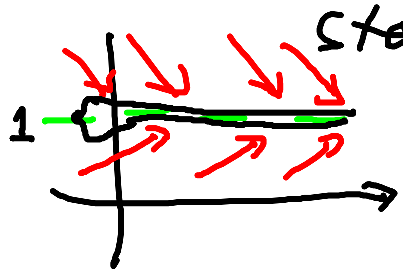
stac. rieš.

$y(x) = 0,\ x \in \mathbb{R}$

nest.

$y(x) = 1,\ x \in \mathbb{R}$

stab.



rovnovážna hodnota

stabilní ekvilibr.

$y_0 = 1$

nestabilní ekvil.

$y_0 = 0$