$$f(a+h) = f(a) + f'(a)h + \frac{1}{2}f''(a)h^2 + \frac{1}{3!}f'''(a)h^3 + \dots$$

$$\cos(a+h) = \cos(a) - \underbrace{\sin(a)}_{0}h - \frac{1}{2}\cos(a)h^2 + \frac{1}{3!}\underbrace{\sin(a)}_{0}h^3 + \frac{1}{4!}\cos(a)h^4 - \dots$$

$a=0$

$$\boxed{\cos(h) = 1 - \frac{1}{2}h^2 + \frac{1}{4!}h^4 - \frac{1}{6!}h^6 + \frac{1}{8!}h^8 - \dots}$$

aprox.

chyba

$$E_x = x - \hat{x}$$

$$x - \hat{x} = E_x$$

$$x = \underbrace{\hat{x}} + \underbrace{E_x}$$

$$\cos(h) \sim 1 \quad \left\| \lim_{h \to 0} \left(\frac{\cos(h)}{1}\right) = 1 \right\|$$

$$E_h = -\frac{1}{2}h^2 + \frac{1}{4!}h^4 - \dots$$

$$E_h = \mathcal{O}(h^2) \quad |E_h| \le c \cdot h^2 \quad \Big\| \quad E_h \sim -\frac{1}{2}h^2 \quad \Big\| \quad E_h = -\frac{1}{2}h^2 + \mathcal{O}(h^4)$$

$\cos(h) \sim 1$

$\cos(h) \sim 1 - \frac{1}{2}h^2$

$E_h = O(h^3)$

$E_h = O(h^4)$ ← lepší!

$$E_h = \underbrace{\frac{1}{4!}h^4}_{0 \cdot h^3} - \frac{1}{6!}h^6 + \dots$$

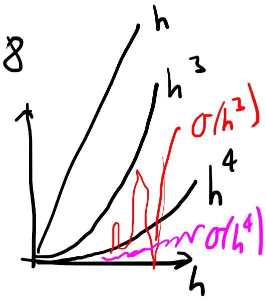
najdi aprox. $\cos(x)$ okolo poč. s chybou $O(h^{10})$

$$\cos(h) = \underbrace{1 - \frac{1}{2}h^2 + \frac{1}{4!}h^4 - \frac{1}{6!}h^6 + \frac{1}{8!}h^8 \dots}$$

f' f'' f'''

$$- \frac{1}{10!}h^{10} + \frac{1}{12!}h^{12} - \dots$$

$O(h^{10})$



$x \rightsquigarrow \hat{x}$, E_x , ε_x $\quad \left\{ E_x = x - \hat{x} \longrightarrow \hat{x} = x - E_x \right.$

$\cos(x) \rightsquigarrow \cos(\hat{x}) \longrightarrow$ chyby ?

$$E_{\cos(x)} = \cos(x) - \cos(\hat{x}) = \cos(x) - \cos(\underbrace{x}_{a} \underbrace{- E_x}_{+h})$$

$$= \cancel{\cos}(x) - \left[\cancel{\cos}(x) - \sin(x)\cdot(-E_x) - \frac{1}{2}\cos(x)(-E_x)^2 + O(E_x^3)\right]$$

$$= -\sin(x)\, E_x + O(E_x^2) \sim -\sin(x)\cdot E_x \quad \left[|E_{\cos(x)}| \leq |E_x| \right.$$

$$\varepsilon_{\cos(x)} = \frac{|E_{\cos(x)}|}{|\cos(x)|} \leq \frac{|\sin(x)|\cdot|E_x|}{|\cos(x)|} = \frac{|\sin(x)|\cdot|x|}{|\cos(x)|}\cdot\frac{|E_x|}{|x|}$$

$$= \overbrace{|x|\cdot|tg(x)|}\cdot\varepsilon_x$$

$x \sim \pi/2$!

$f(x) = \frac{1}{x^2}$ $f'(1) = -2$

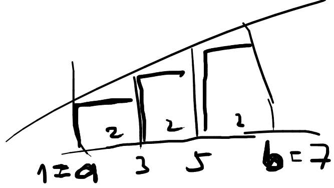
↳ aproximace, dopředná dif., $h = 0.01$.

$$\underline{\underline{f'(1)}} \approx \frac{f(1+h) - f(1)}{h} = \frac{\frac{1}{(1+h)^2} - \frac{1}{1^2}}{h} = \underline{\underline{\frac{\frac{1}{(1.01)^2} - 1}{0.01}}}$$

$$\left[\begin{array}{l} E_h = (-2) - \left[\frac{100}{(1.01)^2} - 100\right] = \dots \\ \varepsilon_x = \frac{|E_h|}{|-2|} = \dots \end{array}\right]$$

$$\boxed{\int_1^7 e^x\,dx}$$

aproximujte daný $\int$ s $n=3$ $\langle 1, 7\rangle$ $\Rightarrow$ $h = 2 = \frac{7-1}{3}$



→ levé obdélníky:

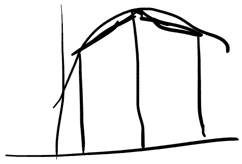
$$I \approx 2\cdot e^1 + 2\cdot e^3 + 2\cdot e^5 = 2\cdot\left[e + e^3 + e^5\right]$$

→ pravé obdélníky

$$I \approx 2\cdot e^3 + 2\cdot e^5 + 2\cdot e^7 = 2\cdot\left[e^3 + e^5 + e^7\right]$$

→ lichoběžníky

$$I \approx 2\cdot\frac{1}{2}(e+e^3) + 2\cdot\frac{1}{2}(e^3+e^5) + 2\cdot\frac{1}{2}(e^5+e^7) = 2\cdot\frac{1}{2}\left[e + 2e^3 + 2e^5 + e^7\right]$$



$$h \cdot \frac{1}{2}\left[f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)\right]$$

řád = 2 $\qquad |E_h| \leq C \cdot h^2$

globální chyba aproximace

záleží, na čem a kde se koukám $\forall \upsilon \, \exists c \dots$

$\int_a^b f(x)\,dx$

$\underline{h} \in (0, \varepsilon) \rightarrow \boxed{C}$

levé obd. $\int_0^1 \cos(x)\,dx$ $\underbrace{h = 0.2}$ $\underbrace{n = 5}$

$\underbrace{E_{0.2} \approx 0.0432}$ $E_5 \approx 0.0432$

<u>a)</u> $E = ?$ <u>počet dělení</u> $\rightarrow \tilde{n} = 10$ $\tilde{n} = 2 \cdot n$

$$E_m \approx c \cdot \left(\frac{1}{m}\right)^1$$

$$E_{2n} \approx c \cdot \left(\frac{1}{2n}\right) = \frac{1}{2} \cdot \underbrace{c \cdot \frac{1}{n}} = \frac{1}{2} E_n$$

$$E_h \approx c \cdot h^1$$

$$\underline{\underline{E_{\frac{h}{2}}}} \approx c \cdot \left(\frac{h}{2}\right) = \frac{1}{2} c \cdot h = \underline{\underline{\frac{1}{2} E_h}}$$

$$\underline{E_{10}} \approx \frac{1}{2} E_5 = \frac{1}{2} \cdot 0.0432 = \underline{0.0216}$$

b) (Počet dělení) Jaký krok aby chyba byla nejvýše $E = 0.001$.

$$\left[\frac{E}{E_h}\right] \cdot h_{old}$$

#1

$E_{0.2} \simeq c \cdot 0.2 \longrightarrow c = \frac{E_{0.2}}{0.2}$

chci $0.001 \simeq c \cdot h \Rightarrow h = \frac{0.001}{c} = \frac{0.001}{\frac{0.0432}{0.2}} = \underbrace{\frac{0.001}{0.0432}}\cdot 0.2$

#2

krok $h \xrightarrow{\text{chci}} \frac{1}{a} h$ víme $E_{\frac{h}{a}} = \frac{1}{a} \cdot E_h$

vím chci vím

$a = \frac{E_h}{E} \rightsquigarrow \boxed{h_{new} = \frac{h}{a} = \frac{E}{E_h} \cdot h_{old}}$

tiich 6. , $h = 0.5$, $E_{0.5} = 0.01760$

a) $\left(\frac{1}{2}h\right)^2$. $p=2$

$$\boxed{E_{\frac{h}{2}} \approx c \cdot \left(\frac{h}{2}\right)^p \approx \frac{1}{2^p} c h^p = \frac{1}{2^p} E_h} \quad \boxed{p = \log_2\left(\frac{E_h}{E_{h/2}}\right)}$$

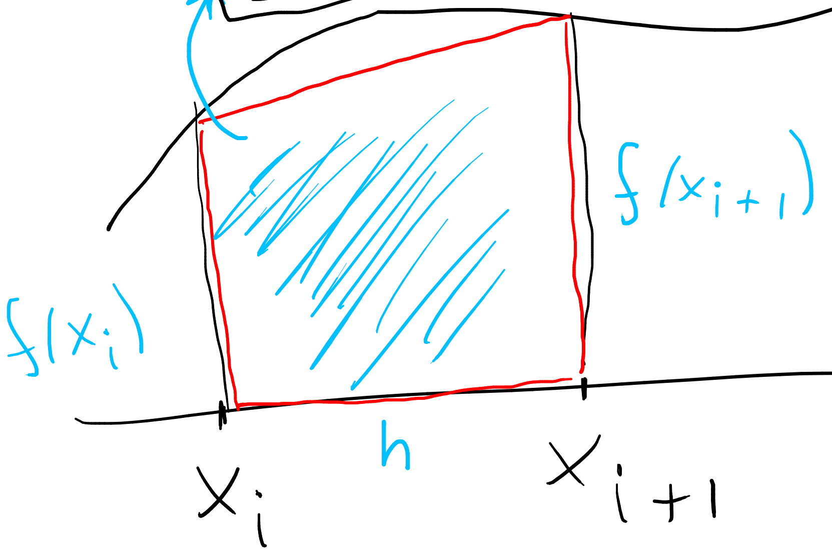
$$E_{\frac{1}{2}0.5} \approx \frac{1}{4} \cdot E_{0.5} = 0.00440$$

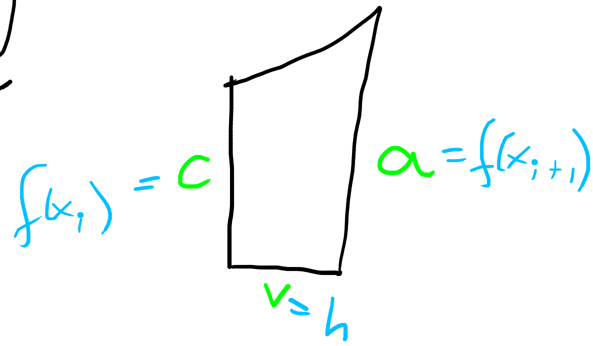
$E_{\frac{h}{a}}$

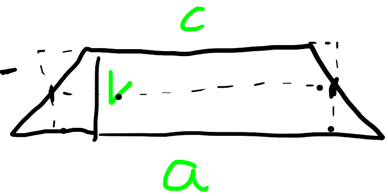
b) chci $E = 0.0001$: $0.0001 = \frac{1}{a^2} E_h$

$$a = \sqrt{\frac{E_h}{0.0001}} \longrightarrow h_{new} = \sqrt{\frac{E_h}{0.0001}} \cdot 0.5$$

$$\boxed{h \cdot \frac{1}{2}\left(f(x_i) + f(x_{i+1})\right)}$$







$$A = v \cdot \frac{a+c}{2}$$

$$h \cdot \frac{1}{2}\left[\circ + 2\circ + \dots\right]$$