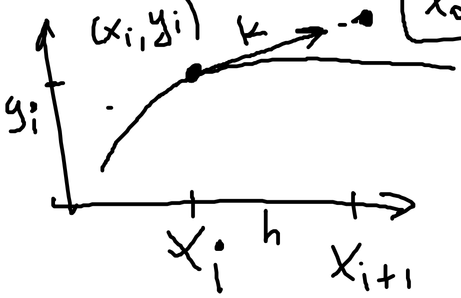
$(x^*, y^*) \rightsquigarrow y'(x^*) = \underbrace{f(x^*, y^*)}$

dáno $n \in \mathbb{N}$

dán $\langle x_0, x_0 + T \rangle$

$h = \frac{T}{n}$



x_0, y_0 dáno

$$x_{i+1} = x_i + h$$

$$y_{i+1} = y_i + \underbrace{f(x_i, y_i)} \cdot h,$$

$$i = 0, \dots, n-1$$

Řád = 1, $|E_h| \le c \cdot h^1 = \underline{\underline{c \cdot h}}$

glob. chyba → $E_n = \max_{i=0,\dots,n} |y(x_i) - y_i|$

$|E_n| \le c \cdot \left(\frac{1}{n}\right)^1 = \frac{c}{n}$

Odhadni řešení $y' = \underbrace{-x(y-1)}$, $y(0) = 2$ Eulerovou metodou na $\langle 0, 5 \rangle$ pro $n = 10$. Spočítej první tři body.

$$h = \frac{5-0}{10} = \frac{1}{2}$$

(0) $x_0 = 0$, $y_0 = 2$

(1) pro $i = 0, \dots, n-1$

$$x_{i+1} = x_i + \frac{1}{2},$$

$$y_{i+1} = y_i \underbrace{- x_i (y_i - 1)} \cdot \frac{1}{2}.$$

1) $(0, 2)$.

2) $x_1 = 0 + \frac{1}{2} = \frac{1}{2} = 0.5$

$k = 0$

$y_1 = 2 + 0 \cdot \frac{1}{2} = 2$

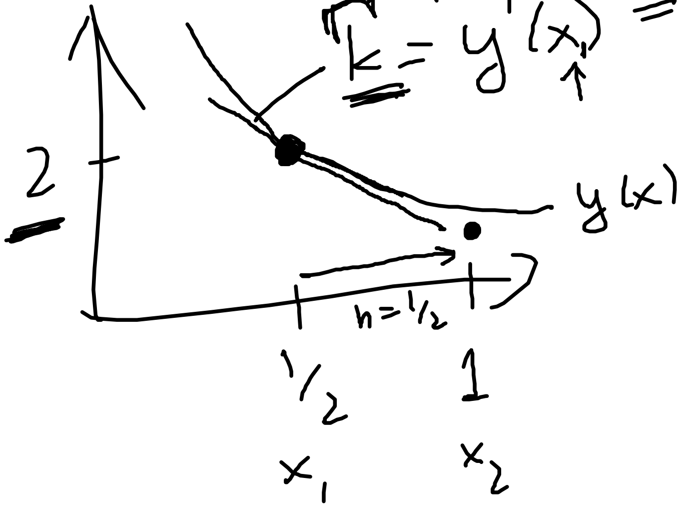
$(\frac{1}{2}, 2)$

$(0.5, 2)$,

3) $x_2 = \frac{1}{2} + \frac{1}{2} = 1$, $k = -\frac{1}{2}(2-1) = -\frac{1}{2}$, $y_2 = 2 - \frac{1}{2} \cdot \frac{1}{2} = \frac{7}{4}$. | $(1, 7/4)$, $(1, 1.75)$

$(\frac{1}{2}, 2) \qquad x_1 = \frac{1}{2} \qquad y_1 = 2 \qquad \| \qquad y' \overset{*}{=} -x(y-1)$

$$k = y'(\underset{\uparrow}{x_1}) = y'(1/2) \overset{*}{=} -\underset{\substack{\uparrow \\ x_1}}{\frac{1}{2}} \cdot (\underset{\substack{\uparrow \\ y_1}}{2} - 1) = -\frac{1}{2}$$



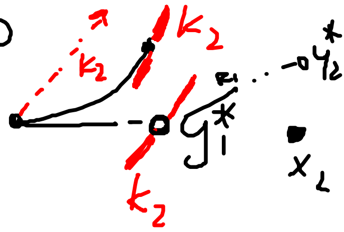
$$y_2 = y_1 + k \cdot h$$

$$y_2 = 2 + \left(-\frac{1}{2}\right) \cdot \frac{1}{2}$$

$$y' = -x(y-1), \quad y(0) = 2$$

$\langle 0, 5 \rangle$, $n = 10$

$h = 1/2$



$\boxed{(0, 2)}$

$$x_1 = \frac{1}{2}$$

$$y_1^* = 2 + \underbrace{0}_{k_1} \cdot \frac{1}{2} = 2$$

$$k_2 = y'(x_1) = -\frac{1}{2} \cdot (2 - 1) = -\frac{1}{2}$$

$\uparrow y_1^*$

$$\underline{\underline{y_1}} = 2 + \left(-\frac{1}{2}\right) \cdot \frac{1}{2} = \underline{\underline{\frac{7}{4}}}$$

$\left(\frac{1}{2}, \frac{7}{4}\right)$

$$x_2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$k_1 = y'\left(\frac{1}{2}\right) = -\frac{1}{2}\left(\frac{7}{4} - 1\right) = -\frac{3}{8}$$

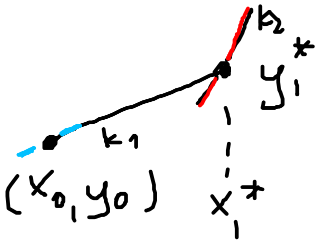
$\uparrow y_1 = \frac{7}{4}$

$$y_2^* = \frac{7}{4} + \left(-\frac{3}{8}\right) \cdot \frac{1}{2} = \frac{28 - 3}{16} = \frac{25}{16}$$

$k_2 = \ldots \quad y_2 = \ldots$

$y' = -x(y-1)$, $y(0) = 2$ na $\langle 0, 5 \rangle$, $n = 10$, $h = \frac{1}{2}$

$\boxed{(0, 2)}$



$x_1 = \frac{1}{2}$

$y_1^* = 2 + \underset{k_1}{0} \cdot \frac{1}{2} = 2$

$k_2 = y'(x_1, \underset{y_1^*}{\ }) = -\frac{1}{2} \cdot (2-1) = -\frac{1}{2}$

$\underline{\underline{y_1}} = 2 + \left(-\frac{1}{2}\right) \cdot \frac{1}{2} = \underline{\underline{\frac{7}{4}}}$

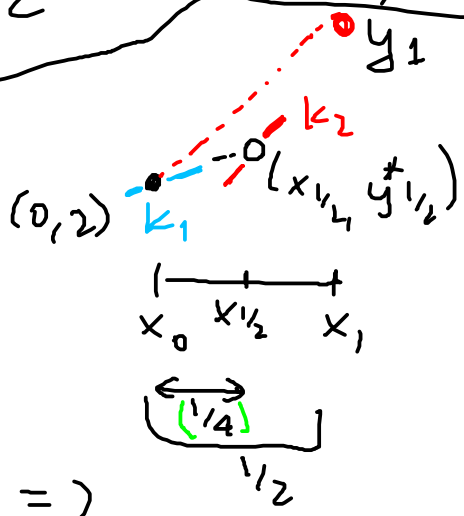
$k = \frac{1}{2}\left(0 + \left(-\frac{1}{2}\right)\right) = -\frac{1}{4}$

$y_1 = 2 + \left(-\frac{1}{4}\right) \cdot \frac{1}{2} = \frac{15}{8}$

$\underline{\underline{\left(\frac{1}{2}, \frac{15}{8}\right)}}$

$\boxed{\text{Heun}}$

$y' = -x(y-1)$, $y(0) = 2$, $\langle 0, 5 \rangle$, $n = 10$, $h = \frac{1}{2}$



$\boxed{(0,2)}$

$x_1 = 0 + \frac{1}{2} = \frac{1}{2}$

$k_1 = y'(0) = 0$

$x_{\frac{1}{2}} = \frac{1}{4}$, $y^*_{\frac{1}{2}} = 2 + 0 \cdot \left(\frac{1}{4}\right) = 2$

$k_2 = y'\left(\frac{1}{4}\right) = -\frac{1}{4}(2-1) = -\frac{1}{4}$

$y_1 = 2 + \left(-\frac{1}{4}\right) \cdot \frac{1}{2} = \frac{15}{8}$.

$\boxed{\text{midpoint}}$

midpoint = RK2

$E_{0.5} \approx 0.01888$

a) $E_{0.25} \approx ?$

$$E_{0.25} = E_{\frac{0.5}{2}} \approx \frac{1}{2^2} E_{0.5} = \frac{1}{4} \cdot 0.01888$$

$$= \underline{0.00472}$$

$0.01888 \approx c \cdot (0.5)^2 \longrightarrow \underline{c} \longrightarrow E_{0.25}$

b) Krok? Chci $E_h = 0.00001$.

chci $0.00001 = c \cdot h^2$

$$\Rightarrow h = \sqrt{\frac{0.00001}{c}} = \dots$$

Řád 2 $\quad |E_h| \le c \cdot h^2$

$E_h \approx c \cdot h^2$

$$E_{\frac{h}{a}} \approx c \cdot \left(\frac{h}{a}\right)^2 = \frac{1}{a^2} c \cdot h^2$$

$$= \frac{1}{a^2} E_h$$

$\underline{0.00001} = E_{\frac{h}{a}} = \frac{1}{a^2} \underline{E_{0.5}}$

$a = \sqrt{\frac{0.01888}{0.00001}}$, nové h: $\frac{0.5}{a}$

Fåd p

$$\boxed{E_{\frac{h}{a}} \approx \frac{1}{a^p} E_h}$$ $$\boxed{E_{p \cdot n} \approx \frac{1}{a^p} E_n}$$

$$a^p = \frac{E_h}{E_{\frac{h}{a}}} \longrightarrow \boxed{p = \log_a\left(\frac{E_h}{E_{\frac{h}{a}}}\right)}$$

$$\underset{\sim}{x} \longrightarrow \underset{\sim}{\hat{x}}, \quad E_x, \ \varepsilon_x \qquad a \ (\in \mathbb{Z})$$

$$\longrightarrow a \cdot x$$

$$\boxed{E_{a\cdot x} = a\cdot x - a\cdot \hat{x}} = a\cdot(x - \hat{x}) = \underline{a\cdot E_x},$$

$$\boxed{\varepsilon_{a\cdot x} = \frac{|E_{a\cdot x}|}{|a\cdot x|} =} \ \dots \qquad = \qquad \cdot\, \varepsilon_x$$

$\underline{x}$ $\quad \hat{x} = x + \underset{\hookrightarrow -Ex}{e}$ $\quad E_x = x - \hat{x}$

$\hat{x} = x - E_x$

$$E_{ax} = a \cdot x - a \cdot \hat{x} = a \cdot x - a \cdot (x + e)$$

$$= -a \cdot e = a \cdot E_x$$

$$\boxed{\varepsilon_{ax} = \varepsilon_x}$$

$$E_{x+y} = E_x + E_y$$

$$|E_{x+y}| \leq |E_x| + |E_y|$$