Řešení $\underbrace{y' + y}_{L} = 4e^x + x$ splňující $y(1) = 2e - \frac{2}{e}$.

Plán
1. obecné řeš. → a) homog. rce
   b) pravá strana (odhad)
2. poč. podm.

1. a) $y' + y = 0$ $\quad \lambda + 1 = 0 \longrightarrow \lambda = -1 \Rightarrow y_h(x) = a \cdot e^{-x}$

b) $b(x) = 4e^x + x$

$4e^x \xrightarrow{\text{základ}} A \cdot e^x \xrightarrow[\lambda = 1]{\lambda = 1 \pm 0i}$ bez korekce $\longrightarrow Ae^x$

$x \xrightarrow{\text{základ}} Bx + C \xrightarrow[\lambda = 0]{\lambda = 0 + 0i}$ bez korekce $\longrightarrow Bx + C$

odhad: $y_p(x) = Ae^x + Bx + C$. Dosadím & vidím

$$L = [Ae^x + Bx + C]' + (Ae^x + Bx + C) = Ae^x + B + Ae^x + Bx + C =$$
$$= 2Ae^x + Bx + (B + C) \overset{?}{=} 4e^x + x + 0$$

$$\left[\begin{array}{l} 2A = 4 \\ B = 1 \\ B + C = 0 \end{array}\right] \Rightarrow \left.\begin{array}{l} A = 2 \\ B = 1 \\ C = -1 \end{array}\right\} \quad y_p(x) = 2e^x + x - 1$$

a) & b): obecné řeš:

$$\boxed{y(x) = 2e^x + x - 1 + ae^{-x}, \; x \in \mathbb{R}}$$

$y(x) = 2e^x + x - 1 + ae^{-x}$ chci $y(1) = 2e - \frac{2}{e}$

$y(1) = \cancel{2e} + \cancel{1-1} + ae^{-1} = \cancel{2e} - \frac{2}{e} \longrightarrow a = -2$

řešení: $\boxed{y(x) = 2e^x + x - 1 - 2e^{-x}, \; x \in \mathbb{R}}$

[illegible] $= e^x \cos(2x)$ 1.

$\big\downarrow$ základ

$A \cdot e^x \cos(2x) + B \cdot e^x \sin(2x)$

$\lambda = 1 + 2i$

$$y' + y = 4e^x + x \qquad \Big|\; y(x) = Ae^x + Bx + C$$

$$[Ae^x + Bx + C]' + (Ae^x + Bx + C) = 4e^x + x$$

$$2Ae^x + Bx + (B+C) = 4e^x + x$$

$$2Axe^x + Bx + (B+C) = 4e^x + x \quad ?$$

$$? \quad Bx + (B+C) = 4e^x + x$$

$y''' - 4y' =$ $\lambda = 0, -2, 2$	$y'' - 4y' + 4y =$ $\lambda = 2$ $(2x)$	$L[y] = \diagup = b(x)$
$x \cdot (Ax+B)\, e^{-2x}$	$(Ax+B)\, e^{-2x}$ ne kor.	$= x e^{-2x}$ $\lambda = -2$
$A e^{2x} \cos(x) + B e^{2x} \sin(x)$ ne kor.	$A \cdot e^{2x} \cos(x) + B e^{2x} \sin(x)$ ne kor.	$= 3e^{2x} \cos(x)$ $\lambda = 2 + 1i$
$(Ax^2 + Bx + C) \cdot x$	$Ax^2 + Bx + C$ ne kor.	$= 5x^2 - 1$ $\lambda = 0$
$(Ax+B)\cos(\pi x) + (Cx+D)\sin(\pi x)$ ne kor.	$(Ax+B)\cos(\pi x) + (Cx+D)\sin(\pi x)$ ne kor.	$= 13x \cos(\pi x)$ $\lambda = \pi i$
$A e^{x} + x \cdot B e^{2x}$	$A \cdot e^{x} + x^2 B e^{2x}$	$= e^{x} - 3e^{2x}$ $\lambda = 1$ ¦ $\lambda = 2$
$x \cdot (A\cancel{x}+B) + C\sin(3x) + D\cos(3x)$	$Ax + B + C\sin(3x) + D\cos(3x)$	$= 2x - \sin(3x)$ $\lambda = 0$ ¦ $\lambda = 3i$

Najdi řešení $y'' - y' = 4\sin(x) - 2x$
splňující $y(0) = 3,\ y'(0) = 0$

1. obec. řeš.: a) hom. $\lambda^2 - \lambda = 0 \iff \lambda(\lambda - 1) = 0 \Rightarrow \lambda = 0, 1$

$$y_h(x) = ae^0 + be^x = a + be^x,\quad x \in \mathbb{R}$$

b) odhad:

$\rightarrow 4\sin(x) \Rightarrow A\sin(x) + B\cos(x) \xrightarrow[\langle \lambda = 0 + 1 \cdot i \rangle]{\lambda = i}$ bez korekce

$\rightarrow -2x \Rightarrow Cx + D \xrightarrow{\lambda = 0}$ překryv (1x) korekce $\Rightarrow (Cx + D) \cdot x = Cx^2 + Dx$

$$y_p(x) = A\sin(x) + B\cos(x) + Cx^2 + Dx \longrightarrow \text{do } L = \ldots = ?$$

$$\underbrace{y'' - y'}_{L} = 4\sin(x) - 2x$$

$$L = [-A\sin(x) - B\cos(x) + 2C] - [A\cos(x) - B\sin(x) + 2Cx + D] =$$

$$= (-A+B)\cdot\sin(x) + (-A-B)\cos(x) - 2Cx + (2C - D) \stackrel{?}{=} 4\sin(x) + 0\cdot\cos(x) + (-2x) + 0$$

$$\begin{bmatrix} -A+B = 4 \\ -A-B = 0 \\ -2C = -2 \\ 2C - D = 0 \end{bmatrix}$$

$\to \oplus\ -2A = 4,\ A = -2$, do (#2), $B = 2$

$\to C = 1$

$\to D = 2$

$$y_p(x) = 2\cos(x) - 2\sin(x) + x^2 + 2x$$

obecné řeš:

$$\boxed{y(x) = 2\cos(x) - 2\sin(x) + x^2 + 2x + a + be^x,\ x \in \mathbb{R}}$$

$y(0) = 3$ $\quad y'(0) = 0$

$$y(x) = 2\cos(x) - 2\sin(x) + x^2 + 2x + a + be^x$$

$$y' = -2\sin(x) - 2\cos(x) + 2x + 2 + be^x$$

chú :

$$\begin{cases} 2 - 0 + 0^2 + 0 + a + b = 3 \\ -0 - 2 + 0 + 2 + b = 0 \end{cases}$$

$$\Rightarrow \begin{cases} a + b = 1 \\ b = 0 \end{cases} \rightarrow a = 1, \quad b = 0$$

Řešení : $y(x) = 2\cos(x) - 2\sin(x) + x^2 + 2x + 1, \quad x \in \mathbb{R}$

1. $y'' - y = 9e^{2x}$
2. $y'' + 3y' + 2y = \sin(x) + 3\cos(x)$
3. $y'' - 4y' + 4y = e^{2x}$
4. $y'' - 4y = -8x$

→ as. řád typického řešení v ∞

↳ $y_h(x)$

↳ $y_p(x)$ stačí odhad!

$y_p + y_h \longrightarrow$ ○

ad 1) $h: \lambda^2 - 1 = 0$, $\lambda = \pm 1$, $y_h = ae^{x} + be^{-x}$

$b(x)$: $y_p = Ae^{2x}$

$y(x) = Ae^{2x} + ae^{x} + be^{-x} \sim Ae^{2x}$ pro $x \to \infty$

ad 2) $h: \lambda^2 + 3\lambda + 2 = (\lambda+2)(\lambda+1) = 0 \Rightarrow \lambda = -1, -2$

$y_h(x) = ae^{-x} + be^{-2x}$

$b(x)$: $y_p(x) = A\sin(x) + B\cos(x)$

$y(x) = A\sin(x) + B\cos(x) + ae^{-x} + be^{-2x} \sim$?

ad 3) $h: (\lambda^2 - 4\lambda + 4) = (\lambda - 2)^2 \overset{!}{=} 0$ $\quad \lambda = 2$ $(2x)$

$y_h(x) = a\underline{e^{2x}} + bxe^{2x}$

$b(x)$: $y_p(x) = A \cdot \underline{e^{2x}} \cdot x^2$ // kor.!

$y(x) = Ax^2e^{2x} + ae^{2x} + bxe^{2x}$

$\sim Ax^2e^{2x}$, $x \to \infty$

ad 4) $h: \lambda^2 - 4 = 0$, $\lambda = \pm 2$ $\quad y_h = ae^{2x} + be^{-2x}$

$b(x)$: $y_p = Ax + B$

$y(x) = Ax + B + ae^{2x} + be^{-2x} \sim ae^{2x}$, $x \to \infty$.

$$\underbrace{y' + y} = 4x$$

$$L[y] = y' + y$$

$$L[Ax + B] = \underbrace{A + B} + Ax \stackrel{?}{=} 4x + \underbrace{0}$$

$$= \sin(2x) + 3\cos(x)$$

$$\downarrow \lambda = 2i \qquad \downarrow \lambda = i$$

$$\boxed{A\sin(2x) + B\cos(2x) + C\cos(x) + D\sin(x)}$$

$$A\sin(x) + B\cos(x) + ae^{x} + be^{-x} \sim ae^{x}$$

$$\downarrow \infty \qquad \downarrow 0$$

$$\underset{\infty}{\underset{\downarrow}{ae^{2x}}} - \underset{0}{\underset{\downarrow}{be^{-2x}}} \sim ae^{2x}$$

$$-ae^{2x} + be^{-2x} \sim -ae^{2x}$$

A

$$\boxed{-Ax+B} \rightarrow \text{-- } x + \text{ --}$$

$$= e^x - x$$

$$(Ae^x + Bx + C) \to y_p$$

$$(Ae^x - Bx - C)$$

$$y(x) = Ae^{2x} + ae^{x} + b \sim Ae^{2x}$$

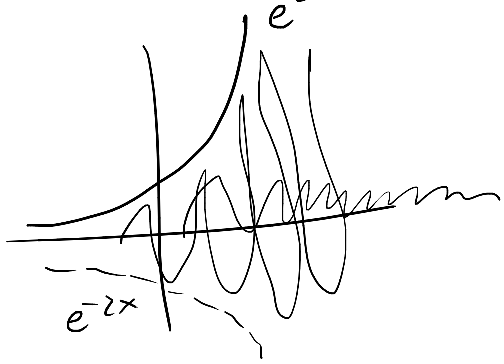
$$\left[y \begin{array}{l} \to \infty \\ \to -\infty \end{array} \right]$$

$$\cancel{y(x) \sim e^{2x}}$$

$$y(x) = O\left(e^{2x}\right)$$

$$|y(x)| \le c \cdot e^{2x}$$

$$y(x) = \Theta\left(e^{2x}\right)$$



$x \cdot \sin(x) + 3\cos(x)$

$$\left\{(Ax+B)\cdot\sin(x) + (Cx+D)\cdot\cos(x)\right\}$$

$\lambda = 2$ $\lambda = 0$ $\lambda = 0$

$$? = e^{2x} + 9x + 18$$

$x^{3} \cdot (Bx+C)$

$Bx^4 + Cx^3$

$$y_p = Ae^{2x} + (Bx+C)$$

$\lambda = 0$

hom

$\boxed{\lambda = 0 \ (3x)}$

$(a + bx + cx^2)$

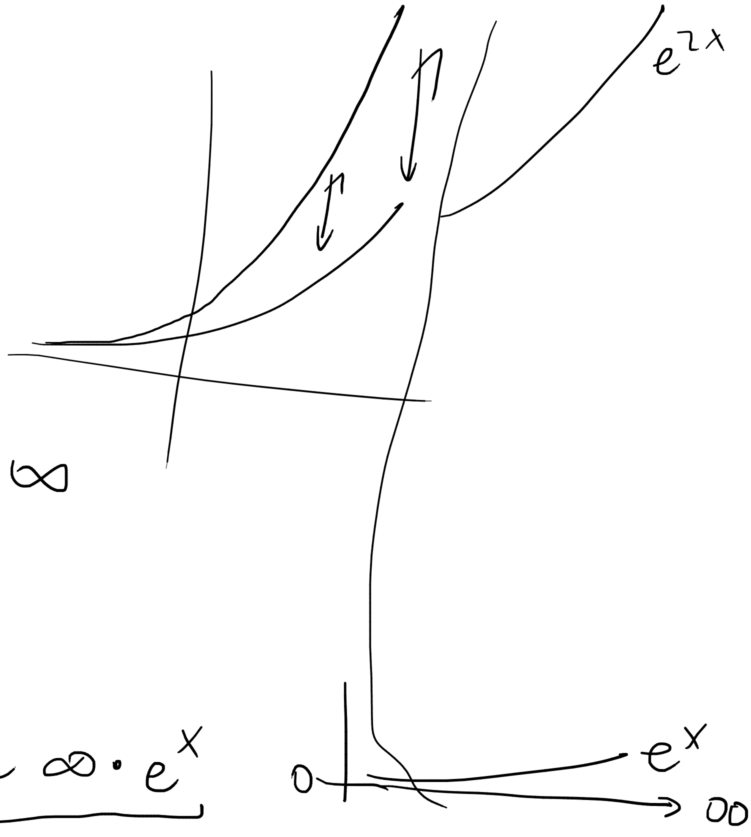
kor? kor?

$$\begin{array}{ccc} e^{2x} & + & \cancel{e^{x}} \\ \downarrow & & \downarrow \\ \boxed{\infty} & & \infty \end{array}$$

$$\lim_{x \to \infty} \frac{e^{2x}}{e^{x}} = \lim(e^{x}) = \infty$$

$$\hookrightarrow \quad \frac{e^{2x}}{e^{x}} \sim \infty$$

$$\underbrace{e^{2x} \sim \infty \cdot e^{x}}$$



1. $L = y'' - y$, $L[Ae^{2x}] = 4Ae^{2x} - Ae^{2x} = 3Ae^{2x} \overset{?}{=} 9e^{2x}$

$\boxed{A = 3}$

4. $L = y'' - 4y$, $L[Ax+B] = 0 - 4(Ax+B) = -4Ax - 4B \overset{?}{=} -8x$

$\boxed{A = 2 \quad B = 0}$

2. $L = y'' + 3y' + 2y$, $L[A\sin(x) + B\cos(x)] = (\underline{-A\sin} \; \underline{-B\cos})$
$+3(\underline{A\cos} - \underline{B\sin}) + 2(\underline{A\sin} + \underline{B\cos}) = (A - 3B)\sin(x) + (3A+B)\cos(x) \overset{?}{=} \sin(x) + 3\cos(x)$

$\left.\begin{array}{l} A - 3B = 1 \\ 3A + B = 3 \end{array}\right\} \oplus \begin{array}{l} A = 1 \\ B = 0 \end{array}$

$\boxed{A = 1 \quad B = 0}$

$$y'' + 2y' = \underset{(1)}{6e^{x}} - 2\underset{(2)}{e^{-2x}} + \underset{(3)}{8\cos(2x)} + \underset{(4)}{4x}$$

$y(0) = 2$

$y'(0) = 2$

a) $\lambda^2 + 2\lambda = 0 \longrightarrow \lambda = 0, -2 \Rightarrow \boxed{y_h(x) = a + be^{-2x}}$

b) $y_p(x) = \underset{\lambda=1}{Ae^{x}} + \underset{\lambda=-2}{x \cdot Be^{-2x}} + \underset{\lambda=2i}{C\cos(2x) + D\sin(2x)} + \underset{\lambda=0}{x \cdot (Ex + F)}$ $\quad Ex^2 + Fx$

$$L[(1)+(2)] = [Ae^{x} - 2Be^{-2x} - 2Be^{-2x} + 4Bxe^{-2x}] + 2[Ae^{x} + Be^{-2x} - 2Bxe^{-2x}] =$$

$$= 3Ae^{x} - 2Be^{-2x} \overset{?}{=} 6e^{x} - 2e^{-2x} \Longrightarrow A = 2, \; B = 1$$

$$L[(3)+(4)] = [-4C\cos(2x) - 4D\sin(2x) + 2E] + 2[-2C\sin(2x) + 2D\cos(2x) + 2Ex + F]$$

$$= \underbrace{(-4C + 4D)}_{8}\cos(2x) + \underbrace{(-4C - 4D)}_{0}\sin(2x) + \underbrace{4E}_{4}x + \underbrace{(2E + 2F)}_{0}$$

$C = -1, \; D = 1 \qquad E = 1 \qquad F = -1$

obecné řešení

$$y(x) = 2e^{x} + x\cdot \underbrace{e^{-2x}}_{e^{-2x} - 2x e^{-2x}} - \cos(2x) + \sin(2x) + x^2 - x + a + be^{-2x}, x \in R$$

p.p.: $\begin{cases} y(0) = \cancel{2} + 0 - 1 + 0 + 0 - 0 + a + b \stackrel{?}{=} \cancel{2} \\ y'(0) = \cancel{2} + 1 - 0 + 0 + 2 + 0 - 1 - 2b \stackrel{?}{=} \cancel{2} \end{cases}$

$$\Leftrightarrow \begin{bmatrix} a + b = 1 \\ -2b = -2 \end{bmatrix} \begin{matrix} \rightarrow a = 0 \\ \rightarrow b = 1 \end{matrix}$$

$$y(x) = 2e^{x} + xe^{-2x} + e^{-2x} - \cos(2x) + \sin(2x) + x^2 - x, x \in R$$

$\uparrow y_h$

$= 7e^{2x} + 16x$ $\boxed{\begin{array}{l} e^{0}\cdot\cos(2x) \\ \lambda = 0 + 2i = 2i \end{array}}$

Ae^{2x} $\quad$ $Bx + C$ $\quad$ $y_p = Ae^{2x} + Bx + C$

$\downarrow L$ $\quad$ $\downarrow L$ $\quad$ $\downarrow L$

$= 7e^{2x}$ $\quad$ $= 16x$

A $\quad$ B, C $\quad$ $= b(x)$

y_p