$$y_1' = 2y_1 - y_2 + 4e^{4x}$$
$$y_2' = -2y_1 + y_2 + 3$$

obecné řeš: $\vec{y}_h$, $\vec{y}_p$

a) homog: $\begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}$ ---- $\begin{bmatrix} y_{1h}(x) = a + be^{3x} \\ y_{2h}(x) = 2a - be^{3x} \end{bmatrix}$ $\underline{\underline{\lambda = 0, 3}}$

b) • $4e^{4x}$, $\lambda = 4$ $\quad Ae^{4x}$

• 3, $\lambda = 0$ $\xrightarrow{kor}$ $B \cdot \underline{x} + C$ $\quad \begin{bmatrix} y_{1p}(x) = Ae^{4x} + Bx + C \\ y_{2p}(x) = De^{4x} + Ex + F \end{bmatrix}$ → dos.

$$\begin{cases} \left[Ae^{4x}+Bx+C\right]' = 2\left(Ae^{4x}+Bx+C\right) - \left(De^{4x}+Ex+F\right) + 4e^{4x} \\ \left[De^{4x}+Ex+F\right]' = -2\left(Ae^{4x}+Bx+C\right) + \left(De^{4x}+Ex+F\right) + 3 \end{cases}$$

$$\begin{cases} 4Ae^{4x} + B = 2Ae^{4x} + 2Bx + 2C - De^{4x} - Ex - F + 4e^{4x} \\ 4De^{4x} + E = -2Ae^{4x} - 2Bx - 2C + De^{4x} + Ex + F + 3 \end{cases}$$

$$\begin{cases} (2A+D)e^{4x} + (-2B+E)x + (B-2C+F) = 4e^{4x} \\ (3D+2A)e^{4x} + (2B-E)x + (2C+E-F) = 3 \end{cases}$$

$$\begin{cases} 2A+D=4 \\ 2A+3D=0 \end{cases} \qquad \begin{cases} -2B+E=0 \\ 2B-E=0 \end{cases} \qquad \begin{cases} B-2C+F=0 \\ 2C+E-F=3 \end{cases}$$

$$Ae^{4x}+Bx+C$$

$$De^{4x}+Ex+F$$

$$\begin{cases} 2A + D = 4 \\ 2A + 3D = 0 \end{cases} \quad \begin{cases} -2B + E = 0 \\ 2B - E = 0 \end{cases} \quad \begin{cases} B - 2C + F = 0 \\ 2C + E - F = 3 \end{cases}$$

$$\downarrow$$

$$2D = -4$$

$$\boxed{\begin{matrix} D = -2 \\ A = 3 \end{matrix}}$$

$$E = 2B \mapsto \begin{cases} B - 2C + F = 0 \\ 2B + 2C - F = 3 \end{cases}$$

$$+ \quad 3B = 3 \Rightarrow \begin{cases} B = 1 \\ E = 2 \end{cases}$$

$$\begin{cases} -2C + F = -1 \\ 2C - F = 1 \end{cases} \rightarrow \left. \begin{matrix} F = 1 \\ C = 1 \end{matrix} \right\} \text{volba}$$

$$y_{1p}(x) = 3e^{4x} + x + 1$$

$$y_{2p}(x) = -2e^{4x} + 2x + 1$$

obecné řeš

$$y_1(x) = 3e^{4x} + x + 1 + a + be^{3x}$$

$$y_2(x) = -2e^{4x} + 2x + 1 + 2a - be^{3x}$$

$x \in R$!

variace

$$y_1(x) = a(x) + b(x)\cdot e^{3x}$$
$$y_2(x) = 2a(x) - b(x)\cdot e^{3x}$$
$\rightarrow$ dos.

$$\begin{cases} [a(x)+b(x)e^{3x}]' = 2\left(a(x)+b(x)e^{3x}\right) - \left(2a(x) - b(x)e^{3x}\right) + 4e^{4x} \\ [2a(x) - b(x)\cdot e^{3x}]' = -2\left(a(x)+b(x)e^{3x}\right) + \left(2a(x) - b(x)e^{3x}\right) + 3 \end{cases}$$

$$\begin{cases} a'(x) + b'(x)e^{3x} + 3\cancel{b(x)e^{3x}} = \cancel{2a(x)} + \cancel{2b(x)e^{3x}} - \cancel{2a(x)} + \cancel{b(x)e^{3x}} + 4e^{4x} \\ 2a'(x) - b'(x)e^{3x} - 3\cancel{b(x)e^{3x}} = -\cancel{2a(x)} - 2\cancel{b(x)e^{3x}} + \cancel{2a(x)} - \cancel{b(x)e^{3x}} + 3 \end{cases}$$

$$\begin{cases} a'(x) + b'(x)e^{3x} = 4e^{4x} \\ 2a'(x) - b'(x)e^{3x} = 3 \end{cases} \quad (+) \quad 3a' = 4e^{4x} + 3$$

$$a' = \frac{4}{3}e^{4x} + 1$$

$$a(x) = \frac{1}{3}e^{4x} + x$$

$$3b'e^{3x} = 8e^{4x} - 3 \rightarrow b' = \frac{8}{3}e^{x} - e^{-3x}$$

$$\Rightarrow b(x) = \frac{8}{3}e^{x} + \frac{1}{3}e^{-3x}$$

$$y_1(x) = a(x) + b(x)\cdot e^{3x}$$
$$y_2(x) = 2a(x) - b(x)\cdot e^{3x}$$

$$\underline{y_1(x)} = \frac{1}{3}e^{4x} + x + \left(\frac{8}{3}e^{x} + \frac{1}{3}e^{-3x}\right)\cdot e^{3x} =$$
$$= \frac{1}{3}e^{4x} + x + \frac{8}{3}e^{4x} + \frac{1}{3} = \underline{3e^{4x} + x + \frac{1}{3}}$$

$$\underline{y_2(x)} = \frac{2}{3}e^{4x} + 2x - \left(\frac{8}{3}e^{x} + \frac{1}{3}e^{-3x}\right)e^{3x} =$$
$$= \frac{2}{3}e^{4x} + 2x - \frac{8}{3}e^{4x} - \frac{1}{3} = \underline{-2e^{4x} + 2x - \frac{1}{3}}$$

var.

odhad:

$$y_1(x) = 3e^{4x} + x + 1 + a + be^{3x} \xrightarrow{a=-2/3} 3e^{4x} + x + 1/3$$
$$y_2(x) = -2e^{4x} + 2x + 1 + 2a - be^{3x}, \; x\in\mathbb{R} \xrightarrow{b=0} -2e^{4x} + 2x - 1/3$$

$$\vec{y}' = A\vec{y} + \vec{b}$$

$$\vec{y}' = A\vec{y}$$

$$\vec{y}_h = Y(x) \cdot \vec{c}$$

var

$$\vec{c}(x)$$

$$\left[Y(x) \cdot \vec{c}(x) \right]' =$$

$$\begin{pmatrix} \cdots \end{pmatrix} \cdot \begin{pmatrix} \ \end{pmatrix} = \ldots$$