

$$\begin{bmatrix} 2x + 4y - 2z = 4 \\ x + y - 2z = 3 \\ -3x + y + 8z = -11 \end{bmatrix} \quad \left( \begin{array}{ccc|c} 2 & 4 & -2 & 4 \\ 1 & 1 & -2 & 3 \\ -3 & 1 & 8 & -11 \end{array} \right)$$

lidová E

- $(\#3) + (\#1) + (\#2) \mapsto (\#3) \quad *$
- $2 \cdot (\#2) - (\#1) \mapsto (\#2) \quad *$
- $\frac{1}{2} \cdot (\#1) \mapsto (\#1)$

$$\left( \begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -2 & -2 & 2 \\ 0 & 6 & 4 & -4 \end{array} \right)$$

- GJM  $(A|\vec{b}) \rightsquigarrow (E|\vec{r})$

$$\left( \begin{array}{ccc|c} 2 & 4 & -2 & 4 \\ 1 & 1 & -2 & 3 \\ -3 & 1 & 8 & -11 \end{array} \right) \begin{array}{l} \text{GJM} \\ (14) \\ \text{ops} \end{array} \left. \begin{array}{l} (\#1) \leftrightarrow (\#2) \\ (\#2) - 2 \cdot (\#1) \mapsto (\#2) \\ (\#3) + 3 \cdot (\#1) \mapsto (\#3) \end{array} \right\} \begin{array}{l} 1 \text{ (7)} \\ 3 \times 2 \\ 1 \text{ (7)} \\ 3 \times 2 \end{array}$$

$$\sim \left( \begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 2 & 2 & -2 \\ 0 & 4 & 2 & -2 \end{array} \right) \begin{array}{l} (11) \\ \text{ops} \end{array} \left. \begin{array}{l} (\#3) - 2 \cdot (\#2) \mapsto (\#3) \\ \frac{1}{2}(\#2) \mapsto (\#2) \\ (\#1) - (\#2) \mapsto (\#1) \end{array} \right\} \begin{array}{l} 1 \text{ (5)} \\ 2 \times 2 \\ 3 \\ 1 \\ 2 \times 1 \\ 3 \end{array}$$

$$\sim \left( \begin{array}{ccc|c} 1 & 0 & -3 & 4 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 2 \end{array} \right) \begin{array}{l} (7) \\ \text{ops} \end{array} \left. \begin{array}{l} -\frac{1}{2}(\#3) \mapsto (\#3) \\ (\#2) - (\#3) \mapsto (\#2) \\ (\#1) + 3(\#3) \mapsto (\#1) \end{array} \right\} \begin{array}{l} 2 \\ 1+1 = 2 \\ 1+2 \\ 3 \end{array}$$

$$\left. \begin{array}{l} x=1 \\ y=0 \\ z=-1 \end{array} \right\} \text{ (check: 32 ops!)} \quad \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array} \right)$$

• GEM,  $\left( \begin{array}{ccc|c} 2 & 4 & -2 & 4 \\ 1 & 1 & -2 & 3 \\ -3 & 1 & 8 & -11 \end{array} \right)$  14 ops  $(\#1) \leftrightarrow (\#2)$   $[GEM \sim \frac{2}{3} \cdot n^3 \sim \oplus(n^3)]$   
 $(\#2) - 2 \cdot (\#1) \mapsto (\#2)$   
 $(\#3) + 3 \cdot (\#1) \mapsto (\#3)$  [n x n A]

$\sim \left( \begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 2 & 2 & -2 \\ 0 & 4 & 2 & -2 \end{array} \right)$   $(\#3) - 2 \cdot (\#2) \mapsto (\#3)$   $1 + 2 \cdot 2 = 5$  (ops)  
 $\sim \left( \begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 2 & 2 & -2 \\ 0 & 0 & -2 & 2 \end{array} \right)$  cell kern  
19 ops [BS: ~n^2]

BS  $\left[ \begin{array}{l} x + y - 2z = 3 \\ 2y + 2z = -2 \\ -2z = 2 \end{array} \right] \rightarrow \begin{array}{l} y = \frac{1}{2}(-2 - 2z) = 0 \\ z = -1 \end{array}$   $\rightarrow x = 3 - y + 2 \cdot z = 1$   
(3 ops) (1 op) (3 ops) (max 5 ops)

CELKEM (GEM + BS) 26 (max 28) cell kern 7 ops

$$\begin{cases} 2x + y + 2z = 1 \\ 4x + 4y + 6z = 2 \\ 2x - 5y - 3z = 0 \end{cases}$$

$(\#2) - 2(\#1) \rightarrow (\#2) \quad 1 + 3 \cdot 2 = 7$   
 $(\#3) - (\#1) \rightarrow (\#3) \quad 1 + 3 \cdot 1 = 4$   
 $(\#3) + 3(\#2) \rightarrow (\#3) \quad 1 + 2 \cdot 2 = 5$

$$\begin{pmatrix} 2 & 1 & 2 & | & 1 \\ 4 & 4 & 6 & | & 2 \\ 2 & -5 & -3 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 2 & | & 1 \\ 0 & 2 & 2 & | & 0 \\ 0 & -6 & -5 & | & -1 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 2 & | & 1 \\ 0 & 2 & 2 & | & 0 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \rightarrow z = -1$$

$$\begin{cases} 2x + y + 2z = 1 \\ 2y + 2z = 0 \\ z = -1 \end{cases} \rightarrow z = -1$$

$x = \frac{1}{2}(1 - y - 2z) = \frac{1}{2}(4) = 2$   
 $y = \frac{1}{2}(-2z) = 1$

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GEM + BS

$$\begin{cases} 2x + y + 2z = 3 \\ 4x + 4y + 6z = 6 \\ 2x - 5y - 3z = 2 \end{cases}$$

$$\begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$$

||||| 5 ops

$$\begin{cases} 2x + y + 2z = 3 \\ 2y + 2z = 0 \\ z = -1 \end{cases} \rightarrow x = 2, y = 1, z = -1$$

6 ops

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \underline{3} & 1 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \underline{-1} & 0 & 1 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ \underline{-2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \cdot \begin{pmatrix} 2 & 1 & 2 \\ 4 & 4 & 6 \\ 2 & -5 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ \underline{-7} & 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 4 & 4 & 6 \\ 2 & -5 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}^{-1} \cdot \underbrace{\begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}}_U$$

A

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -3 & 1 \end{pmatrix} = L$$

$$A = L \cdot U$$

$$\underbrace{\begin{pmatrix} 2 & 1 & 2 \\ 4 & 4 & 6 \\ 2 & -5 & -3 \end{pmatrix}}_L \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -3 & 1 \end{pmatrix}}_L \cdot \underbrace{\begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}}_A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -3 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$$

$$\left[ \begin{array}{l} u = 3 \\ 2u + v = 6 \\ u - 3v + w = 2 \end{array} \right] \rightarrow \begin{array}{l} u = 3 \\ v = 6 - 2u = 0 \\ w = 2 - u + 3v = -1 \end{array}$$

$$\underbrace{\begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}}_L \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{cases} 2x + y + 2z = 3 \\ 2y + 2z = 0 \\ z = -1 \end{cases}$$

$$\downarrow$$

$$\underline{x=2 \quad y=1 \quad z=-1}$$

$$A = \begin{pmatrix} 4 & -3 & -5 \\ -1 & 2 & 1 \\ 2 & -2 & -3 \end{pmatrix} \rightarrow \text{vl. \&tilde{e}i\&tilde{e}la}$$

$$\lambda^3 - 3\lambda^2 - \lambda + 3 = 0$$

$$\lambda = 1, -1, \underline{3}$$

↑     ↑     ↑



$$\rho(A) = 3$$

$$\|A\|_{\infty} = \max(4+3+5, 1+2+1, 2+2+3) = 12$$

$$\|A\|_1 = \max(4+1+2, 3+2+2, 5+1+3) = 9$$

$$\|A\|_F = \sqrt{73} \approx 8.54$$

$$\|A\|_2 \approx 8.46$$

$$\| \|A\| \| < 3.004$$