$$\left[\begin{array}{l} x + 2y + 2z = 1 \\ x + y + z = 1 \\ x - y + z = 5 \end{array}\right] \xrightarrow{\text{iterace}} \left[\begin{array}{l} x = 1 - 2y - 2z \\ y = 1 - x - z \\ z = 5 - x + y \end{array}\right]$$

iniciace $\vec{x}_0 = (2, 0, 0)$.

- **Jacobi**

$$x_{k+1} = 1 - 2y_k - 2z_k$$
$$y_{k+1} = 1 - x_k - z_k$$
$$z_{k+1} = 5 - x_k + y_k$$

$$\vec{x}_{k+1} = B \cdot \vec{x}_k + \vec{c}$$

$$B_J = \begin{pmatrix} 0 & -2 & -2 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \qquad \rho(B_J) > 1$$

řešení $x = 1$
$y = -2$
$z = 2$

	$x = 1 - 2y - 2z$	$y = 1 - x - z$	$z = 5 - x + y$
$k=0$	2	0	0
$k=1$	$x = 1$	$y = -1$	$z = 3$
$k=2$	$x = -3$	$y = -3$	$z = 3$
$k=3$	$x = 1$	$y = 1$	$z = 5$
$k=4$	$x = -11$	$y = -5$	$z = 5$
	$\downarrow$ 1 ?	$\downarrow$ -2 ?	$\downarrow$ 2 ?

$x = 2$
$y = 0$
$z = 0$

① $x = 1$
$y = 0$
$z = 4$

② $x = -7$
$y = 4$
$z = 16$

③ $x = -3 \xrightarrow{?} 1$
$y = 24 \xrightarrow{?} -2$
$z = 68 \xrightarrow{?} 2$

Pesimismus.

$$\boxed{\begin{aligned} x &= 1 - 2y - 2z \\ y &= 1 - x - z \\ z &= 5 - x + y \end{aligned}}$$

$$\left[\begin{aligned} x_{k+1} &= 1 - 2y_k - 2z_k \\ y_{k+1} &= 1 - x_{k+1} - z_k \\ z_{k+1} &= 5 - x_{k+1} + y_{k+1} \end{aligned}\right]$$

- Gauss - Seidel

$$\left[\begin{aligned} x_3 &= -3 \\ y_3 &= 24 \\ z_3 &= 68 \end{aligned}\right]$$

$$\begin{cases} x_{k+1} = 1 - 2y_k - 2z_k \\ y_{k+1} = 1 - x_{k+1} - z_k \\ z_{k+1} = 5 - x_{k+1} + y_{k+1} \end{cases} \rightsquigarrow$$

$$x_{k+1} = 1 - 2y_k - 2z_k$$

$$y_{k+1} = 1 - (1 - 2y_k - 2z_k) - z_k = 2y_k + z_k$$

$$z_{k+1} = 5 - (1 - 2y_k - 2z_k) + (2y_k + z_k) = 4 + 4y_k + 3z_k$$

$$\vec{x}_{k+1} = B \cdot \vec{x}_k + \vec{c}$$

$$B_{GS} = \begin{pmatrix} 0 & -2 & -2 \\ 0 & 2 & 1 \\ 0 & 4 & 3 \end{pmatrix} \to \rho(B_{GS}) = ? \quad \begin{pmatrix} >1 \\ \text{asi} \end{pmatrix}$$

$$\approx 4.562$$

$$\|B_{GS}\|_1 = 8. \quad -\lambda^3 + 5\lambda^2 - 2\lambda = 0$$

$$\lambda = 0, \ \lambda = \tfrac{1}{2}(5 \pm \sqrt{17})$$

$$\begin{bmatrix} 2x + y + z = 2 \\ x + 2y + z = -1 \\ x + y + 4z = 7 \end{bmatrix}$$

$$\left.\begin{matrix} 2 > 1 + 1 \quad ? \\ 2 > 1 + 1 \quad ? \end{matrix}\right]$$

$$\left. 4 > 1 + 1 \right]$$

$$\begin{bmatrix} x = 1 - \frac{1}{2}y - \frac{1}{2}z \\ y = -\frac{1}{2} - \frac{1}{2}x - \frac{1}{2}z \\ z = \frac{7}{4} - \frac{1}{4}x - \frac{1}{4}y \end{bmatrix}$$

$$\vec{x}_0 = (2, 0, 0)$$

rešení $1, -2, 2$

J	x	y	z
	$x = 1 - \frac{1}{2}y - \frac{1}{2}z$	$y = -\frac{1}{2} - \frac{1}{2}x - \frac{1}{2}z$	$z = \frac{7}{4} - \frac{1}{4}x - \frac{1}{4}y$
$k=0$	2	0	0
$k=1$	$x = 1$	$y = -3/2$	$z = \frac{5}{4}$
$k=2$	$x_2 = \frac{9}{8}$	$y_2 = -\frac{13}{8}$	$z_2 = \frac{15}{8} \approx 2$
$k=3$	$x_3 = \frac{7}{8}$ $\to 1$	$y_3 = -2$ $\to -2$	$z_3 = \frac{15}{8} \approx 2$

$x = 2$

$y = 0$

$z = 0$

GS

$x = 1 - \frac{1}{2}y - \frac{1}{2}z$

$y = -\frac{1}{2} - \frac{1}{2}x - \frac{1}{2}z$

$z = \frac{7}{4} - \frac{1}{4}x - \frac{1}{4}y$

---

(1) $x = 1$

$y = -1$

$z = \frac{7}{4}$

---

(2) $x_2 = \frac{5}{8} \longrightarrow 1$?

$y_2 = -\frac{27}{16} \longrightarrow -2$ (?)

$z_2 = \frac{129}{64} \longrightarrow 2$!

Nadějné!

$$\begin{bmatrix} x + 2y + 2z = 1 \\ x + y + z = 1 \\ x - y + z = 5 \end{bmatrix} \quad \begin{bmatrix} x + y + z = 1 \\ x + 2y + 2z = 1 \\ x - y + z = 5 \end{bmatrix}$$

- Gauss-Seidel, 2 iterace, $\vec{x}_0 = (2,0,0)$

$$\begin{bmatrix} x = 1 - y - z \\ y = \frac{1}{2} - \frac{1}{2}x - z \\ z = 5 - x + y \end{bmatrix}$$

$x = 2$
$y = 0$
$z = 0$

1. $x = 1$
   $y = 0$
   $z = 4$

2. $x = -3 \longrightarrow 1$?
   $y = -2$
   $z = 6 \longrightarrow 2$?

$$x_{k+1} = 1 - y_k - z_k$$
$$y_{k+1} = \frac{1}{2} - \frac{1}{2}x_{k+1} - z_k$$
$$z_{k+1} = 5 - x_{k+1} + y_{k+1}$$

$\boxed{GS}$

relaxace ($\underline{\omega}$)

$$x_{k+1} = \omega\left(1 - y_k - z_k\right) + (1-\omega)\, x_k$$
$$y_{k+1} = \omega\left(\frac{1}{2} - \frac{1}{2}x_{k+1} - z_k\right) + (1-\omega)\, y_k$$
$$z_{k+1} = \omega\left(5 - x_{k+1} + y_{k+1}\right) + \underbrace{(1-\omega)}\, z_k$$

GS

SOR

$\boxed{\text{Zk:}}$ $\vec{x}_k \overset{?}{\longleftrightarrow} \cancel{A}\vec{x}_k + \vec{b}$

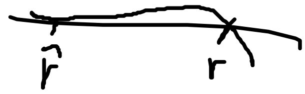
chci $\left[\varphi_\lambda(x) = \lambda \varphi(x) + (1-\lambda)x \longrightarrow \varphi'_\lambda(x_0) = 0 \right]$

$\cancel{\lambda = \frac{\omega}{\omega}}$ $\cancel{\omega\lambda^n + \ldots}$

ted: $\vec{x}_{k+1} = B\vec{x}_k + \vec{c}$ $\longrightarrow$ chci $\cancel{\rho(\omega B + (1-\omega)E_n) < 1}$

↳ relax: $\vec{x}_{k+1} = \omega B \vec{x}_k + (1-\omega)E_n \vec{x}_k + \vec{c}$

$f(x) = 0$? $\longrightarrow$ $\hat{r}$



poznám $\underbrace{\hat{r} - r}$

pomocí $|f(\hat{r}) - 0|$ malé ?

$\longrightarrow$ $\underbrace{\text{test tří}}$

$\underbrace{A\vec{x} + \vec{b} = \vec{0}}$ // $\underbrace{A\vec{x} = \vec{b}}$ $\longrightarrow$ $\hat{\vec{x}}$

$A\hat{\vec{x}} + \vec{b} \overset{?}{\longleftrightarrow} \vec{0}$ | $A\hat{x} \overset{?}{\longleftrightarrow} \vec{b}$ malý rozdíl / $\longrightarrow$ není info

te~~st tří~~