1a) $\rightarrow$ $y_h(x) = ae^x + be^{4x}$, $x \in \mathbb{R}$

$\rightarrow$ $y_p(x) = A\cos(x) + B\sin(x) + C\,x\cdot e^x$

$L = \underbrace{(3A-5B)}_{0}\cos(x) + \underbrace{(5A+3B)}_{34}\sin(x) \underbrace{-3C}_{-3} e^x$

obecné řeš: $y(x) = xe^x + 5\cos(x) + 3\sin(x) + ae^x + be^{4x}$, $x \in \mathbb{R}$

p.p. $\boxed{y(x) = xe^x + 5\cos(x) + 3\sin(x) + e^x,\ x \in \mathbb{R}}$

1b) $\boxed{\text{viz web.}}$

2a) $3y' = -\frac{2}{x^3 y^2} \longrightarrow x \neq 0,\ y \neq 0$

$$\int 3y^2 \, dy = \int -\frac{2}{x^3} \, dx$$

$$y^3 = \frac{1}{x^2} + C$$

$$\cancel{y = \sqrt[3]{\frac{1}{x^2}} + C}$$

obecné řeš.: $y(x) = \sqrt[3]{\frac{1}{x^2} + C},\ x \neq 0,\ \frac{1}{x^2} + C \neq 0$

$y(1) = -1$

$\sqrt[3]{1 + C} = -1$

$1 + C = (-1)^3$

$1 + C = -1$

$C = -2$

p.p.: $y(x) = \left(\frac{1}{x^2} - 2\right)^{1/3},\ x \in \left(\frac{1}{\sqrt{2}}, \infty\right)$

$-\frac{1}{\sqrt{2}}$ 0 $\frac{1}{\sqrt{2}}$ 1

2b1) $\begin{pmatrix} -6 & 1 \\ -8 & 0 \end{pmatrix} \to \lambda = -2, \; -4$

$$-2 \to \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \qquad -4 \to \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\vec{y} = a \begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{-2x} + \dots$

$$\begin{bmatrix} y_1(x) = a e^{-2x} + b e^{-4x}, \\ y_2(x) = 4a e^{-2x} + 2b e^{-4x}, \quad x \in \mathbb{R} \end{bmatrix}$$

(0,0) je stabilní, protože

- $Re(\lambda) < 0 \quad \forall \lambda$
- $\vec{y} \to \vec{0}$ pro $x \to \infty \quad \forall \vec{y}$

2b2) $\rightarrow y_h(x) = Ce^{3x}$

$\rightarrow$ V $\quad y_p(x) = C(x)\, e^{3x}$

$$L = C'(x)\, e^{3x} = 2e^{x}$$

$$C'(x) = 2e^{-2x}$$

$$\boxed{C(x) = -e^{-2x}} \quad +C$$

$$y_p(x) = -e^{-2x}\cdot e^{3x} = -e^{x}$$

obecné řeš:

$$\boxed{y(x) = -e^{x} + Ce^{3x}, \ x \in \mathbb{R}}$$

$$y' - 3y = 0$$

$\rightarrow \lambda - 3 = 0$

$\rightarrow \dfrac{dy}{dx} = 3y$

$$\int \frac{dy}{y} = \int 3\, dx$$

$$\ln|y| = 3x + C$$

$$|y| = e^{3x+C}$$

$$y = \underbrace{\pm e^{C}}\cdot e^{3x}$$

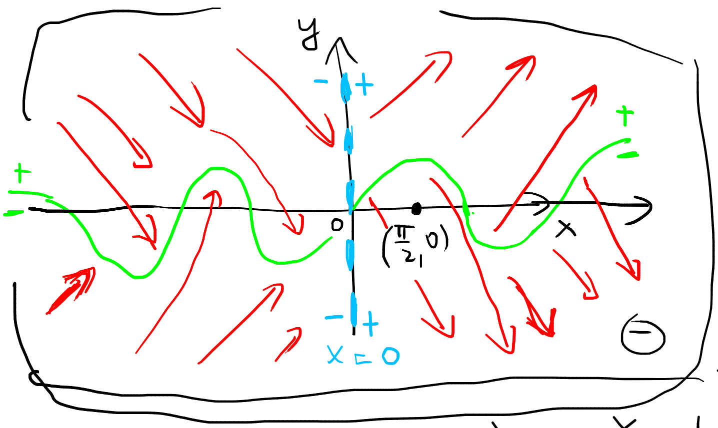
$$y_h = De^{3x}$$

3a) $y' = \underbrace{x}_{} \cdot \underbrace{(y - \sin(x))}_{}$ → neex ? X

→ $= 0$? → $\boxed{x = 0}$

→ $y - \sin(x) = 0$

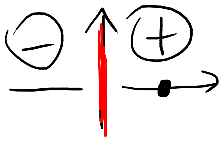
$\boxed{y = \sin(x)}$

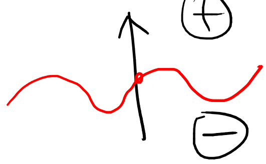


$\boxed{\text{stac. res.}}$ neex

$y' = 0$? $\forall_x$

$y' = \underbrace{e^x - 1}_{}$	$y' = y - \sin(x)$
$y' = 0$ $e^x - 1 = 0$ $e^x = 1$ $\boxed{x = 0}$	$y' = 0$ $y - \sin(x) = 0$ $\boxed{y = \sin(x)}$





$$y' - x \cdot y = x \cdot \check{s}in(x)$$ → odhad ? **NE** **lin**

$$\underbrace{y' - x}_{a(x)} \cdot y = 0$$

↳ separace $y_h(x) = C \cdot e^{x^2/2}$

var: $C'(x) e^{x^2/2} = x \cdot \check{s}in(x)$

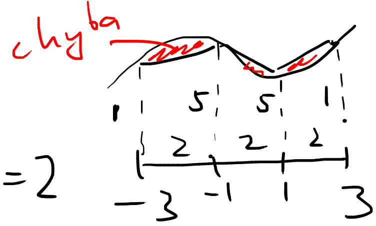
$$\boxed{C(x) = \int x e^{-x^2/2} \check{s}in(x) dx}$$

3 b) $y' + \frac{1}{x} \cdot y = \frac{1}{x}$ $\qquad y' = \frac{1}{x} - \frac{1}{x} y$

- separace : ano , neboť $y' = \frac{1}{x}(1-y)$.
- odhad : ne ← ~~nemá~~ není konst. koef.
  ← není spec. PS.
  - lin. : ✓
  - konst. koef. : ✗
  - spec. PS : ✗
- variace : ano
  - lin : ✓
  - umím y_h → konst. koef. / → 1. řád ✓

3c) viz web..

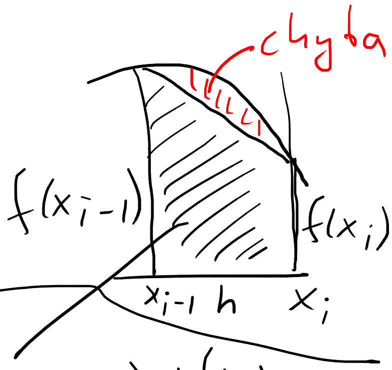
4a) $\int_{-3}^{3} \frac{10}{x^2+1}\,dx$, lichob, $n=3$



$$h = \frac{3-(-3)}{3} = 2$$

$$I \approx 2\cdot\frac{1+5}{2} + 2\cdot\frac{5+5}{2} + 2\cdot\frac{5+1}{2}$$

$$= 2\cdot\frac{1}{2}\left[1 + 2\cdot 5 + 2\cdot 5 + 1\right]$$



4b)

$$A_i = h\cdot\frac{f(x_{i-1})+f(x_i)}{2}$$

$$\sum A_i =$$