1 Lab 1 – February 17, 2022

1.1 Which conditions must be satisfied by sets A, B, C to guarantee that:

a) $(A \setminus C) \setminus B = A \setminus (C \setminus B)$;

b) $A \cap (B \cup C) = (A \cap B) \cup C$;

c) $A \cup (B \oplus C) = (A \cup B) \oplus (A \cup C)$;

d) $A \setminus (B \cup C) = (A \setminus B) \setminus C$;

e) $(A \cap B) \setminus C = A \cap (B \setminus C)$;

f) $A \cap (B \setminus C) = (A \setminus C) \cap B$;

g) $A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C)$.

1.2 Decide whether or not the following assertions hold; give arguments for your answers.

a) $A \times B = \emptyset$ if and only if $A = \emptyset$ or $B = \emptyset$.

b) For all sets A, B we have $A \times B = B \times A$.

c) For all sets A, B, C the following holds: If $B \subseteq C$, then $A \times B \subseteq A \times C$.

d) If $A \times B \subseteq A \times C$, then $B \subseteq C$.

e) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

f) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

g) $(B \oplus C) \times A \ = (B \times A) \oplus (C \times A)$.

h) If $A \oplus B = A \oplus C$, then $B = C$.

i) $A \setminus (B \oplus C) = (A \setminus B) \oplus (A \setminus C)$.

1.3 List all the subsets of the set $\{1, 2, 3, 4\}$. How many are there?

1.4 There are 200 students in a school, 140 of them can speak French, 80 students can speak German, and 20 students do not know either of these languages. How many students speak both languages?

1.5 Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$. List all mappings from A into B. How many maps are there? Which of them are injective? Which of them are surjective?

1.6 Find an example of a mapping $f : \mathbb{N} \longrightarrow \mathbb{N}$ such that

a) f is injective but not surjective,
b) f is surjective but not injective,
c) f is injective and surjective,
d) f is neither injective nor surjective.

1.7 Show that the rule

$$(m, n) \mapsto 2^m(2n + 1) - 1 \quad (m, n \in \mathbb{N})$$

defines an injective mapping of the set $\mathbb{N} \times \mathbb{N}$ onto $\mathbb{N}$.

1.8 Show that the set of all binary words is countable. (A binary word is a finite sequence of 0's and 1's.)

1.9

a) Show that any two non-empty open intervals (a, b) and (c, d) of real numbers have the same cardinality.
b) Show that the set $\mathbb{R}$ and the set of all positive real numbers $(0, \infty)$ have the same cardinality.