2 Lab 2 – February 24, 2022

2.1 Write the following relations on a set A as sets of ordered pairs:

a) A is the set of all subsets of the set $\{1, 2\}$, relation R is "to be a proper subset". This means that for $X, Y \in A$ we have $X\,R\,Y$ if and only if $X \subseteq Y$ and $X \neq Y$.

b) $A = \{2, 4, 5, 8, 45, 60\}$, R is the relation of divisibility; i.e. $m\,R\,n$ if and only if m divides n.

2.2 A relation R on a closed interval $A = [0, 4]$ is given by:

$$x\,R\,y \quad \text{if and only if} \quad x^2 + y^2 + 7 \leq 4x + 4y.$$

Decide a) whether $2(\,R \circ R\,)2$ and b) whether $0\ (R^{-1} \circ R)\ 3$.

2.3 A relation R on a closed interval $A = [0, 1]$ is given by: $x\,R\,y$ if and only if $y = 2|x - \frac{1}{2}|$. Sketch in a plane (as a set of ordered pairs) the relations R, R^{-1} and $R \circ R^{-1}$.

2.4 Give the properties of the following relations on the set of all natural numbers $\mathbb{N}$:

a) $m\,R\,n$ if and only if m divides n;

b) $m\,R\,n$ if and only if $m + n \geq 50$;

c) $m\,R\,n$ if and only if $m + n$ is even;

d) $m\,R\,n$ if and only if $m \cdot n$ is even;

e) $m\,R\,n$ if and only if $m = n^k$ for some $k \in \mathbb{N}$;

f) $m\,R\,n$ if and only if $m + n$ is a multiple of 3;

g) $m\,R\,n$ if and only if $m > n$.

2.5 In the following examples S is a relation on a set A and x, y are elements of set A. Decide whether S is reflexive, symmetric, antisymmetric, transitive. Is it an equivalence, an order relation?

a) A is the set of all complex numbers, $x\,S\,y$ if and only if $|x| = |y|$.

b) A is the set of all complex numbers, $x\,S\,y$ if and only if $|x| < |y|$.

c) A is the set of all real numbers, $x\,S\,y$ if and only if $x - y$ is a rational number.

d) A is the set of all triangles of a given plane, two triangles are related in S if and only if they are congruent.

e) A is the set of all triangles of a given plane, two triangles are related in S if and only if they are similar.

f) A is the set of all subsets of a set B, two subsets X, Y of the set B are related in S if and only if they have the same cardinality; i.e., if and only if there exists an injective mapping of X onto Y.

2.6 Given two relations R and S from a set A into a set B. Decide whether the following is true:

a) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$;

b) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$.

2.7 Given two relations R and S on a set A. Decide whether it is true:

a) If R and S are reflexive, then so is $R \circ S$.

b) If R and S are symmetric, then so is $R \circ S$.

c) If R and S are antisymmetric, then so is $R \circ S$.

d) If R and S are transitive, then so is $R \circ S$.