3 Lab 3 – March 3, 2022

3.1 Show that the relation of divisibility $|$ is a partial order on the set of all natural numbers $\mathbb{N}$.

(We have $n \mid m$ if and only if there is a $k \in \mathbb{N}$ such that $m = k \cdot n$.)

3.2 Draw the Hasse diagram of the following partial odered set $(A, \sqsubseteq)$. where A is the set of all divisors of 60 and $\sqsubseteq$ is the relation of divisibility.

3.3 Consider the set A of all binary words together with the following relation

$$u \sqsubseteq v \quad \text{iff } u \text{ is a prefix of } v.$$

Show that $(A, \sqsubseteq)$ is a poset.

3.4 Prove by mathematical induction that for every $n \geq 1$ it holds that

$$1 \cdot 2 + 2 \cdot 3 + \ldots + n \cdot (n+1) = \frac{n\,(n+1)\,(n+2)}{3}.$$

3.5 Prove by mathematical induction that for every $n \geq 1$ we have

$$n\,! \geq 2^{n-1}.$$

3.6 Let $x \geq -1$ be a real number. Prove using mathematical induction that for every $n \geq 1$ we have

$$(1+x)^n \geq 1 + n\,x.$$

3.7 Prove using mathematical induction that for every $n \geq 1$ the number $6 \cdot 7^n - 2 \cdot 3^n$ is divisible by 4.

(Note that the same statement can also be proved directly using properties of divisibility by 4.)