4 Lab 4 – March 10, 2022

4.1 Prove using mathematical induction that for every $n \geq 1$ we have

$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n}.$$

4.2 Using mathematical induction prove that for every $n \geq 2$ we have

$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{1}{2}$$

4.3 Using mathematical induction prove that for every $n \geq 2$ we have

$$\sum_{k=1}^{n} \frac{1}{\sqrt{k}} > \sqrt{n}$$

4.4 Let $r \geq 2$. Using mathematical induction prove that for every $n \geq 1$ and every natural numbers $a_0, a_1, \ldots, a_{n-1}$ where $a_i < r$ for $i = 0, \ldots, n-1$, it holds that

$$a_0 + a_1 r + a_2 r^2 + \ldots + a_{n-1} r^{n-1} < r^n.$$

4.5 Using mathematical induction prove that for the Fibonacciho sequence $\{F(n)\}_{n=0}^{\infty}$ it holds:

$$F(n+2) \geq \left(\frac{3}{2}\right)^n$$

for all $n \geq 0$.

Fibonacciho sequence is defined by: $F(0) = 0$, $F(1) = 1$, and $F(n+2) = F(n+1) + F(n)$.

4.6 Using mathematical induction prove that for all $n \geq 1$ and arbitrary real numbers x_i, $i = 1, \ldots n$, it holds that

$$|\sum_{i=1}^{n} x_i| \leq \sum_{i=1}^{n} |x_i|.$$

4.7 Using mathematical induction prove that there is a constant d, $d > 0$, for which

$$\sum_{i=1}^{n} 3^i \leq d \cdot 3^n.$$