## 8 Lab 8 - April 7, 2022

8.1 On the set of all real numbers $\mathbb{R}$ we define an operation $\circ$ by

$$
x \circ y=\frac{x+y}{2} .
$$

Decide whether ( $\mathbb{R}, \circ$ ) forms a semigroup.
8.2 Given a non empty set $A$. Define an operation $\circ$ on $A$ by

$$
x \circ y=x \quad \text { for every } x, y \in A
$$

Decide whether $(A, \circ)$ is a semigroup and whether it has a neutral element.
8.3 Given a non empty set $U$. Consider the set $\mathcal{P}(U)$ of all its subsets. On $A=\mathcal{P}(U)$ define two binary operations: intersection $\cap$ and union $\cup$. Decide whether $(A, \cap)$ and $(A, \cup)$ form semigroups, and whether they have a neutral element.
8.4 On the set $A=\mathbb{Q} \backslash\{0\}$ an operation $\star$ is given by

$$
x \star y=\frac{1}{3} x y .
$$

Show that $(A, \star)$ is a group.
8.5 On the set $A=\mathbb{Q} \backslash\{0\}$ an operation $\circ$ is given by

$$
x \circ y=\frac{1}{\frac{1}{x}+\frac{1}{y}} .
$$

Decide whether $(A, \circ)$ is a semigroup, and whether it has a neutral element.
8.6 Calculate $5^{676}$ in $\left(\mathbb{Z}_{306}, \cdot, 1\right)$ and use it to find all elements $x \in \mathbb{Z}_{306}$ for which

$$
5^{676} \cdot x=3(2 x+1) \quad \text { in }\left(\mathbb{Z}_{306}, \cdot, 1\right) .
$$

8.7 $\quad$ In $\mathbb{Z}_{148}$ the following equation with parameter $p$ is given

$$
p x-5^{509}=9 x+7 .
$$

a) Find all parameters $p$ for which the equation above has a unique solution.
b) Solve the equation above for three such parameters (from a)).

