## 8 Lab 8 – April 7, 2022

8.1 On the set of all real numbers  $\mathbb{R}$  we define an operation  $\circ$  by

$$x \circ y = \frac{x+y}{2}.$$

Decide whether  $(\mathbb{R}, \circ)$  forms a semigroup.

**8.2** Given a non empty set A. Define an operation  $\circ$  on A by

$$x \circ y = x$$
 for every  $x, y \in A$ .

Decide whether  $(A, \circ)$  is a semigroup and whether it has a neutral element.

**8.3** Given a non empty set U. Consider the set  $\mathcal{P}(U)$  of all its subsets. On  $A = \mathcal{P}(U)$  define two binary operations: intersection  $\cap$  and union  $\cup$ . Decide whether  $(A, \cap)$  and  $(A, \cup)$  form semigroups, and whether they have a neutral element.

8.4 On the set  $A = \mathbb{Q} \setminus \{0\}$  an operation  $\star$  is given by

$$x \star y = \frac{1}{3}xy.$$

Show that  $(A, \star)$  is a group.

8.5 On the set  $A = \mathbb{Q} \setminus \{0\}$  an operation  $\circ$  is given by

$$x \circ y = \frac{1}{\frac{1}{x} + \frac{1}{y}}.$$

Decide whether  $(A, \circ)$  is a semigroup, and whether it has a neutral element.

**8.6** Calculate  $5^{676}$  in  $(\mathbb{Z}_{306}, \cdot, 1)$  and use it to find all elements  $x \in \mathbb{Z}_{306}$  for which

$$5^{676} \cdot x = 3(2x+1)$$
 in  $(\mathbb{Z}_{306}, \cdot, 1)$ .

8.7 In  $\mathbb{Z}_{148}$  the following equation with parameter p is given

$$p\,x - 5^{509} = 9\,x + 7.$$

- a) Find all parameters p for which the equation above has a unique solution.
- b) Solve the equation above for three such parameters (from a)).