9 Lab 9 – April 14, 2022

Midterm test

9.1 Calculate 5^{676} in $(\mathbb{Z}_{306}, \cdot, 1)$ and use it to find all elements $x \in \mathbb{Z}_{306}$ for which

$$5^{676} \cdot x = 3(2x+1)$$
 in $(\mathbb{Z}_{306}, \cdot, 1)$.

9.2 In \mathbb{Z}_{148} the following equation with parameter p is given

$$p\,x - 5^{509} = 9\,x + 7.$$

- a) Find all parameters p for which the equation above has a unique solution.
- b) Solve the equation above for three such parameters (from a)).
- **9.3** Given a monoid $(\mathbb{Z}_{124}, \cdot)$ and a parametric equation with parameter *a* where

$$ax + 2 = 5(x + a) - 1.$$

- 1. Give the number of parameters $t \in \mathbb{Z}_{124}$ for which the above equation has precisely one solution. Justify your answer.
- 2. For three such parameters solve the above equation.

9.4 Given a group $(\mathbb{Z}_{11}^{\star}, \cdot, 1)$ of all invertible elements of $(\mathbb{Z}_{11}, \cdot, 1)$. Show that it is a cyclic group. Find at least one generating element. How many generating elements $(\mathbb{Z}_{11}^{\star}, \cdot, 1)$ has?

9.5 Given a group $(\mathbb{Z}_{26}^{\star}, \cdot, 1)$ of all invertible elements of $(\mathbb{Z}_{26}, \cdot, 1)$. Show that it is a cyclic group. Find at least one generating element. How many generating elements $(\mathbb{Z}_{26}^{\star}, \cdot, 1)$ has?