## 9 Lab 9 - April 14, 2022

## Midterm test

9.1 Calculate $5^{676}$ in $\left(\mathbb{Z}_{306}, \cdot, 1\right)$ and use it to find all elements $x \in \mathbb{Z}_{306}$ for which

$$
5^{676} \cdot x=3(2 x+1) \quad \text { in }\left(\mathbb{Z}_{306}, \cdot, 1\right)
$$

9.2 $\quad$ In $\mathbb{Z}_{148}$ the following equation with parameter $p$ is given

$$
p x-5^{509}=9 x+7 .
$$

a) Find all parameters $p$ for which the equation above has a unique solution.
b) Solve the equation above for three such parameters (from a)).
9.3 Given a monoid $\left(\mathbb{Z}_{124}, \cdot\right)$ and a parametric equation with parameter $a$ where

$$
a x+2=5(x+a)-1 .
$$

1. Give the number of parameters $t \in \mathbb{Z}_{124}$ for which the above equation has precisely one solution. Justify your answer.
2. For three such parameters solve the above equation.
9.4 Given a group $\left(\mathbb{Z}_{11}^{\star}, \cdot, 1\right)$ of all invertible elements of $\left(\mathbb{Z}_{11}, \cdot, 1\right)$. Show that it is a cyclic group. Find at least one generating element. How many generating elements ( $\mathbb{Z}_{11}^{\star}, \cdot, 1$ ) has?
9.5 Given a group $\left(\mathbb{Z}_{26}^{\star}, \cdot, 1\right)$ of all invertible elements of $\left(\mathbb{Z}_{26}, \cdot, 1\right)$. Show that it is a cyclic group. Find at least one generating element. How many generating elements ( $\mathbb{Z}_{26}^{\star}, \cdot, 1$ ) has?
