

10 Lab 10 – April 21, 2022

10.1 Given a group $(\mathbb{Z}_{26}^*, \cdot, 1)$ of all invertible elements of $(\mathbb{Z}_{26}, \cdot, 1)$. Show that it is a cyclic group. Find at least one generating element. How many generating elements $(\mathbb{Z}_{26}^*, \cdot, 1)$ has?

10.2 Given a monoid $(\mathbb{Z}_{124}, \cdot)$ and a parametric equation with parameter a where

$$a x + 2 = 5(x + a) - 1.$$

1. Give the number of parameters $t \in \mathbb{Z}_{124}$ for which the above equation has precisely one solution. Justify your answer.
2. For three such parameters solve the above equation.

10.3 Solve the following difference equation

$$5a_n = a_{n+1} + 6a_{n-1}, \quad n \geq 1, \quad a_1 = 9, a_2 = 21.$$

10.4 Solve the following difference equation

$$a_{n+2} = 2a_{n+1} - a_n, \quad n \geq 0, \quad a_0 = 1, a_1 = 2.$$

10.5 Solve the following difference equation

$$a_{n+3} + 3a_{n+2} - 4a_n = 0, \quad n \geq 0, \quad a_0 = 1, a_1 = 1, a_2 = 2.$$

10.6 Solve the following difference equation

$$a_n = -2na_{n-1} + 3n(n-1)a_{n-2}, \quad a_0 = 1, a_1 = 2 \tag{1}$$

using the substitution $a_n = n!b_n$.