

12 Lab 12 – May 5, 2022

12.1 Decide whether the following holds:

- $2^{n+1} \in \Theta(2^n)$,
- $2^{\frac{n}{2}} \in \Theta(2^n)$.

12.2 Given functions

$$4^n, n!, n^{\frac{1}{\ln n}}, e^n, \ln(n!), \ln \ln n, n \lg n, \ln(n^{\ln n}), \sqrt{n}, \frac{1}{n}.$$

Sort these functions in f_1, f_2, \dots , such that $f_i \in \mathcal{O}(f_{i+1})$ where $i = 1, 2, \dots$. State all the pair of functions f, g for which $f \in \Theta(g)$.

12.3 Prove or disprove:

- If for two non negative functions we have $f(n) \in \mathcal{O}(g(n))$ then $2^{f(n)} \in \mathcal{O}(2^{g(n)})$.
- For every non negative function $f(n)$ we have $f(n) \in \mathcal{O}(f(n)^2)$.

12.4 Given the function $f(n) = \sum_{i=1}^n \frac{1}{i}$. Find a simplest function $g(n)$ for which $f(n) \in \Theta(g(n))$.

12.5 Find an upper bound for

$$\sum_{k=1}^n \frac{k}{3^k}.$$

Hint: Use a suitable geometric series.

12.6 Prove the following assertion:

Given three non negative functions $f(n)$, $g(n)$, and $h(n)$ for which there exists $n_0 \in \mathbb{N}$ such that for every $n \geq n_0$

$$g(n) \leq f(n) \leq h(n).$$

Assume that $g(n) \in \Omega(k(n))$ and $h(n) \in \mathcal{O}(k(n))$. Then $f(n) \in \Theta(k(n))$.