3 Asymptotic growth of functions

3.1 Solving homework from last tutorial

1) Prove or disprove: *Assume that for non negative functions $f(n)$ and $g(n)$ we have $f(n) \in \Theta(h(n))$ and $g(n) \in \omega(h(n))$. Then*

$$f(n) + g(n) \in \Theta(g(n)).$$

2) State the asymptotic growth:

   a) $\sum_{i=0}^{n} (7i^3 + 3i^2 + 16)$.

   b) $\sum_{i=1}^{n} (5^i + i^{100})$.

3.2 Using derivation trees show that the function $T(n)$ which is give on natural numbers by the following reccurence

$$T(n) = 3\,T(\frac{n}{3}) + 1,\ T(1) = 1$$

it holds $T(n) \in \mathcal{O}(n)$.

3.3 Strassen algorithm for fast multiplication of matrices. Our aim is to multiply two square matrices of order 2^k. The matrix is divided into four matrices of order 2^{k-1} and the multiplication is performed according to the following equations

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} S_1 + S_2 - S_4 + S_6 & S_4 + S_5 \\ S_6 + S_7 & S_2 - S_3 + S_5 - S_7 \end{pmatrix} \tag{1}$$

kde

$$\begin{array}{lcllcl} S_1 & = & (B - D) \cdot (G + H) & S_5 & = & A \cdot (F - H) \\ S_2 & = & (A + D) \cdot (E + H) & S_6 & = & D \cdot (G - E) \\ S_3 & = & (A - C) \cdot (E + F) & S_7 & = & (C + D) \cdot E \\ S_4 & = & (A + B) \cdot H & & & \end{array} \tag{2}$$

Calculate time requirements for Strassen algorithm.

3.4 A function $T(n)$ is given on natural number by the following recurrence. Find it asymptotic behaviour. Justify you answer (i.e. either state the theorem you have used, or solve directly).

1. $T(n) = 4\,T(\frac{n}{2}) + n\log_2 n$, $T(1) = 1$.
2. $T(n) = 9\,T(\frac{n}{3}) + n^2$, $T(1) = 1$.
3. $T(n) = 30\,T(\frac{n}{25}) + n^{\frac{3}{2}} \log_2 n$, $T(1) = 1$.

3.5 A function $T(n)$ is given on natural number by the following recurrence. Find it asymptotic behaviour. Justify you answer (i.e. either state the theorem you have used, or solve directly).

1. $T(n) = T(n-1) + n^c$, $T(1) = 1$,
   a kde $c \geq 1$ is a constant from $\mathbb{R}^+$.
2. $T(n) = T(\sqrt{n}) + 1$, $T(1) = 1$. (Hint: Use substitution $T(2^m) = S(m)$.)
3. $T(n) = T(n-1) + 2$, $T(1) = 1$.

3.6 Using the Master Theorem solve the following recurrences. If it cannot be solved by Master Theorem, justify why. State Master theorem.

1. $T(n) = 5\ T(\frac{n}{4}) + n$
2. $T(n) = 2T(\frac{n}{2}) + n\log n$

Problems to be solved before next tutorial

3.7 A function $T(n)$ is given on natural number by the following recurrence. Find it asymptotic behaviour. Justify you answer (i.e. either state the theorem you have used, or solve directly).

1. $T(n) = 2\,T(\frac{n}{3}) + 1$, $T(1) = 1$.
2. $T(n) = 4\,T(\frac{n}{3}) + n^{\frac{3}{2}} \log_2 n$, $T(1) = 1$.

3.8 A function $T(n)$ is given on natural number by the following recurrence. Find it asymptotic behaviour. Justify you answer (i.e. either state the theorem you have used, or solve directly).

1. $T(n) = T(n-1) + c^n$, $T(1) = 1$, a kde $c > 1$ je konstanta z $\mathbb{R}^+$.