Exercise 1. Given three non-negative functions $f(n)$, $g(n)$ and $h(n)$. Prove or disprove:

If $f(n) \in \mathcal{O}(h(n))$ *and* $g(n) \in \Omega(h(n))$ *then*

$$f(n) + g(n) \in \Theta(h(n)).$$

Exercise 2. Given three non-negative functions $f(n)$, $g(n)$ and $h(n)$. Prove or disprove:

If $f(n) \in o(h(n))$ *and* $g(n) \in \Omega(h(n))$ *then*

$$f(n) + g(n) \in o(h(n)).$$

Exercise 3. Given three non-negative functions $f(n)$, $g(n)$ and $h(n)$, where $f(n)$ is non-decreasing. Prove or disprove:

If $g(n) \in \mathcal{O}(h(n))$ *then*

$$f(n) \cdot g(n) \in \mathcal{O}(f(n) \cdot h(n)).$$

Exercise 4. For each of the given functions find a function which is the simplest possible and which grows asymptotically the same as f. Give reasons to your assertions.

1. $f(n) = 100n^2 + n \cdot (\lg n)^4$.
2. $f(n) = 4^{\lg n} + 2^n$.

Exercise 5. State the asymptotic growth. Justify.

1. $\sum_{i=1}^{n} \frac{1}{i^2}$.
2. $\sum_{i=1}^{n} (\lg n)^2$.

Exercise 6. A functon $T(n)$ on the set of natural numbers by a recursion. rekurentním vztahem. State its asymptotic growth. Justify.

1. $T(n) = 2\,T(\frac{n}{3}) + n$, $T(1) = 1$.
2. $T(n) = 4\,T(\frac{n}{3}) + n^{\frac{1}{2}} \lg n$, $T(1) = 1$.

Results

1. Exercise 1. It does not hold.
2. Exercise ě. It does not hold.
3. Exercise š. It holds.
4. Exercise 4. 1. $f(n) \in \Theta(n^2)$. 2. $f(n) \in \Theta(2^n)$.
5. Exercise 5. 1. $\sum_{i=1}^{n} \frac{1}{i^2} \in \Theta(1)$. 2. $\sum_{i=1}^{n} (\lg n)^2 \in \Theta(n \lg^2 n)$.
6. Exercise 6. 1. $T(n) \in \Theta(n)$. 2. $T(n) \in \Theta(n^{\log_3 4})$.