Homeworks for KAS 2011/12

1. **Problem.** Construct Turing machines:
   (a) M_1 that accepts $L_1 = \{ww^R | w \in \{0,1\}^*\}$, i.e. the word and its reverse,
   (b) M_2 constructs for each input $w \in \{0,1\}^*$ the word ww, i.e. $(\epsilon, q_0, w) \vdash^\star (\epsilon, p, ww),\ p \in F$.

   In each case prove that your Turing machine does what is required.

2. **Problem.** Prove the following statement: *For every nondeterministic Turing machine M there exists a deterministic Turing machine N suct h that*
   $$L(M) = L(N).$$

3. **Problem.** Prove:
   (a) *Every Turing machine can be simulated by a computer program.*
   (b) *For every computer program P there exists a Turing machine that simulates P.*

4. **Problem.** Show that CNF SAT $\lhd_p$ 3-CNF SAT.

5. **Problem.** Prove that the problem of 2-CNF SAT is in the class $\mathcal{P}$.

   Hint. For a formula φ in 2-CNF construct a directed graph $G = (V, E)$ by:

   - Let $\{u_1, \dots, u_n\}$ are logical variables of φ, Put
   $$V = \{u_1, \neg u_1, u_2, \neg u_2, \dots, u_n, \neg u_n\}.$$
   - For each clause $l_1 \wedge l_2$ there are two directed edges in E; $(\neg l_1, l_2)$ and $\neg l_2, l_1)$. (In other words, reprezent each clause $l_1 \wedge l_2$ by two implications $\neg l_1 \to l_2$ and $\neg l_2 \to l_1$).

   Prove that the formula φ is satisfiable iff each logical variable u_i and its negation $\neg u_i$ belong to different components of strong connectivity.

   Use the followint proposition for a polynomial algorithm decideing whether φ is satisfiable.

6. **Problem.** Show that Hamiltonian circuit problem $\lhd_p$ Hamiltonian path problem.

7. **Problem.** Show that SubsetSum problem $\lhd_p$ partition problem.

8. **Problem.** Show that k colorability problem is $\mathcal{NP}$-complete for every $k > 3$.

9. **Problem.** Consider the following problem: Given undirected graphs G and H. Does there exist a subgraph of G isomorphic to H?

   Show that the problem above is $\mathcal{NP}$ complete.