13th SEFI

European Seminar on Mathematics in Engineering Education

Buskerud University College,
Kongsberg, Norway,
June 11-14, 2006

PROCEEDINGS

SEFI
European Society for Engineering Education

MWG
Mathematics Working Group

BUSKERUD
University College

EDITORS

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Impressions of the 13th SEFI Mathematics Working Group Seminar on Mathematics in Engineering Education

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Overview

The 2006 seminar of the SEFI Mathematics Working Group took place at Buskerud University College, Kongsberg, Norway on 11-14 June. The seminar was attended by about 45 delegates from 15 countries and included presented papers covering the main themes of:

- Innovative Methods of Teaching
- Assessment
- Using the Web in Teaching

There were also working sub-groups plus plenary discussions on:

- The Bologna Agreement
- Current Practice in Assessment
- The Use of Technology in Teaching

The SEFI-MWG holds seminars every two years. It began its work in the 1980s as a forum for exchanging views on how mathematics was taught to engineers across Western Europe. In the 1990s academic from Eastern Europe became actively involved and the Group produced a Core Curriculum that advised institutions on the mathematical components of professional engineering undergraduate programmes. This included not only analysis and calculus, traditionally associated with engineering education, but also the emerging disciplines of linear algebra, probability and statistics and discrete mathematics. By 2002, it had taken the curriculum forward into a more structured and hierarchical form, inclusive of level of study Core Zero (prerequisites), Cores One and Two, together with emphasis on engineering specialisation and learning outcomes.

The Group has also carefully monitored new and effective ways of delivering teaching, maintaining close attention to the evolution of good practice of using computer technology in both teaching and assessment. These have been the subject of many presented papers at the regular seminars, the one in Kongsberg being no exception. It is important to note that, since the SEFI-MWG was formed in 1982, computer technology from the student standpoint has moved forward from programming at a terminal in a remote access environment to one where many students have full internet access, are regular users of on-line communication such as email, blogs and chat rooms and have a well-developed ability to search and investigate. Their teachers, unfortunately, note others changes too; such as a declining lack of student knowledge and traditional drill and practice on entry to an engineering programme. These might cause anxiety and inhibit the formation of higher-level concepts.

However, in 2006, delegates report that many institutions continue to deliver their teaching and assessment in the traditional but tried and tested manner of lectures, supported tutorials and written examinations.

The Proceedings

The first day, Monday, was devoted to Innovative Methods of Teaching. Topics included computer associated ventures such as a relational geometry based tool-environment in which mechanical engineers are trained to utilise their mathematically rich programmes, but also included non-computer based but well focused mathematics support for student engineers aiming to make good deficiencies in traditional background. Also, a full discussion in both of the working sub-groups and the plenum took place on the implications of the Bologna Agreement (see below).

On Tuesday, on the Assessment of Learning Outcomes, there were presentations and discussion on the use of computerised tests to reinforce learning and also excellent examples of how to assess concepts with multiple-choice questions delivered in a more traditional manner. Student group working is becoming more common in a number of institutions and ways in which it can be assessed will always be controversial. Does one, for example, allow individual students to assess the contributions of each other, i.e. use peer group assessments, and, if so, to what measure? In the discussions that followed, many delegates noted the considerable changes that had taken place over the past 30 years in both student preparedness and the curriculum whilst observing the unswerving reliance put upon a final assessment in the form of a traditional examination. In spite of innovations in coursework and groupwork, the end-of-unit examination survives as the principal benchmark of achievement for most academic subjects throughout European institutions, even though variations of both written and written/oral forms are commonly used. The debate is open as to whether the traditional examination succeeds in eliciting a reliable reading of the student's knowledge, understanding, capacity to reason, work through or prove. However, it is quantifiable within specified bounds in terms of effort to set, mark and administer; also, and most importantly, it enjoys a high level of confidence in society at large well beyond the bounds of any institution.

On Wednesday, attention focused on the Use of Technology and the Web in Teaching. Without doubt, the development of the Web in popular culture has opened up enormous opportunities to those who wish to learn and communicate. The advantages to distance learning and assessment, if the material is good enough, are boundless. Many of the delegates use the Web and email as a principal form of communication with students and most of the courses taught by delegates have web support. Some have adopted e-learning environments such as Blackboard. A sense of proportion needs to be kept in that students need to learn, understand, be skilled and enjoy the experience without undue threat. There is some evidence, for example, that ill-considered computer aided teaching and assessment can cause frustration and anxiety, and there is ample evidence that students can learn from well-written manuscript notes in .pdf form, so notes need not always be typed. These accepted, technology looks set to make more strides into the traditional teaching and assessment environment.

Implications of the Bologna Agreement

A new and emerging task for the SEFI-MWG is to consider the implications of Bologna for the curriculum and the likely effects this would have on the mathematical education of engineers. The Agreement calls for Bachelor programmes of 180 European Credits (ECTS), taken over about 3 years or 6-7 semesters, followed by 120 ECTS up to masters level (taking about 2 years or 4 semesters). The decision to opt for this was political and dictated by the aim of transference in academic study between European institutions. This might be good in principle but might be inapplicable and inconsistent with the programmes that some institutions offer. However, other institutions are moving this way and some countries (for example, Belgium) are more committed than others. Delegates were worried about what Bachelors qualifications might come to mean, noting that there could be a wide variety of levels and that mathematics might be put under pressure. Some however commented that Bologna could work if given time to bed in, but there may be a need to distinguish between those Bachelor programmes that naturally lead on to a Master programme and research and others that constituted an exit pathway, i.e. an end of formal study.

Later, SEFI-MWG delegates were asked to go away and address the mathematical requirements of a Bachelor programme. This seems very sensible but there are two considerations to be accounted for initially. Firstly, the start point of mathematical study seems to get lower with every passing year. This would need to be rationalised in terms of Core Zero and Core One of the SEFI-MWG 2002 Core Curriculum. Also, the endpoint of study would bcome before the end of Core Two: this too would need to be rationalised in terms of the type of programme and engineering discipline. What the group might do, is define curriculua for Bachelor Type A (i.e. proceeding to masters) and Bachelor Type B (terminating). Less academically able students might opt for Type B and it might be more open as to the start level of what such a mathematics programme should be.

Acknowledgements

The SEFI-MWG is most grateful to Odd Bringslid and his staff at Buskerud University College in Kongsberg for the organisation of such an active and enjoyable event. Kongsberg enjoyed marvellous weather throughout the Seminar with 30-degree temperatures and midsummer light. It was thus an interesting contrast to listen to an

organ recital in its possibly unique three-storey church and to have the conference dinner deep in a silver mine at a temperature of a mere five degrees.

Next Time in Loughborough

The 14th SEFI-MWG Seminar is planned for April 2008 at Loughborough University in England. It will be a shared event with the Mathematical Education of Engineers Conference, sponsored by the Institute of Mathematics and Its Applications. This has taken place every three years in Loughborough since 1994. At a time when the mathematical background of engineering students is in a continual state of flux, as is the mathematical education of engineers and the needs of Bologna, there is an on-going requirement for the Curriculum to evolve. Nonetheless, there is strong evidence that innovative methods of teaching and assessment appear to be rising in response. The joint event promises to be most memorable.

Contributions

The mathematical expertise of mechanical engineers

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1. Introduction

The mathematical education of engineers has two major goals: It should enable students to understand and use (and maybe develop on their own) mathematical models that are used in application subjects such as Engineering Mechanics or Control Theory. Secondly, students should have a mathematical basis for their future professional life. For achieving the first goal, as a mathematics lecturer one has to investigate the use of mathematics in scripts or text books used by application colleagues. This might be tedious for a mathematician not familiar with the application area (like the author) but, in principle, there are no insuperable barriers. The other goal is far more nebulous because it is much harder to obtain information on the mathematical expertise that is necessary in the daily life of an engineer (for an overview of studies on workplace mathematics see Bessot and Ridgway 2000). Moreover, this will depend on the kind of job, since there is no such thing as “the” engineer. With respect to civil engineers, Kent and Noss (2003) tried to shed more light on the mathematical part of the practice by visiting construction firms, interviewing engineers and managers and joining engineers in their daily life. Although this gives direct access to practical work, the method is very time-consuming for both the researcher and the engineer who must explain his work. Moreover, for an outsider it is hard to gain a good understanding in a short time. Therefore, the research presented in this contribution uses a different approach. We hired students during their last (8th) semester of study, gave them a typical design task, and studied the mathematical components of their work. In this contribution we report on the types of mathematical thinking that were exhibited in the students’ work. We also outline some consequences for the mathematical education of mechanical engineers. Moreover, we comment on the limitations of the approach.

2. Method of investigation

Even within the area of mechanical engineering one can find a variety of different job profiles for construction, production and sales engineers (to name but a few). Moreover, there are engineers working at the forefront of research and development and those doing the “normal” practical design work. Since our graduates mainly find jobs as “normal” design engineers, we concentrated on this job profile. An application colleague who worked for several years in the car industry set up tasks which he considered as comprising normal design work which might be encountered in industry. The tasks we have used so far are described briefly in the next section. We paid two students in their last semester to work on each task for about 100 hours. The

application colleague acted as a mentor playing the role a group or project leader would have in industry. The students use the tools they would work with if they worked on the task in industry: for CAD these are the programs SolidEdge® and Pro/Engineer®; for computing stiffness and eigenfrequencies, they used a version of ANSYS® which is adapted to the needs of a design engineer (i.e. not showing all the details a computational engineer would be interested in).
The students are required to make notes on their work (reasoning and decisions made). Based on their notes and the CAD files they deliver, the students are interviewed and they demonstrate how they used the programs. This is recorded with the screen (and audio) recording software Camtesia®. If necessary, there are additional interviews to clarify the thinking processes. This allows for deeper probing into the thoughts of the students that enable them to work effectively and efficiently on the tasks. The documents and the interviews are used for detecting overt and hidden forms of mathematical thinking and concepts.

It is also necessary to take into account the restrictions of our approach. The students definitely do not have a real workplace environment. There is no project team, and – although they are paid – commitment might be different. They also do not modify a larger existing construction as is often the case in industry. It is hoped that the feedback provided by the application colleague involved in the project resembles the one an engineer in industry would receive from his environment. Secondly, there is the question of representativeness: are the tasks representative of industrial tasks and do the students adequately represent the work of at least junior engineers in industry? Again, the judgement of the application colleague is crucial here.

3. Practical tasks

Last year (2005) two students worked on a task which consisted of designing a support for an ABS (Automated Braking System) in a car. The installation space and

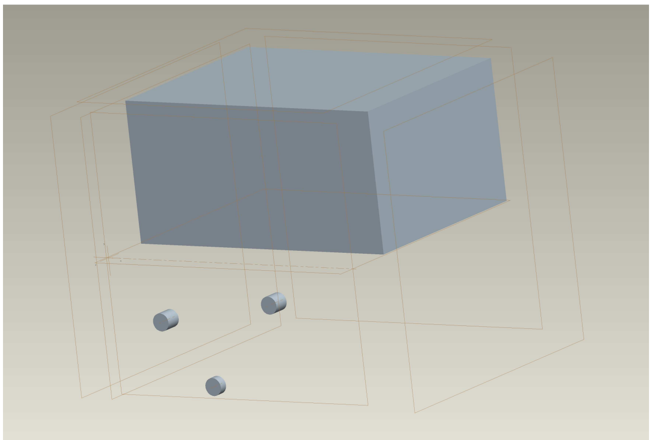

Figure 1: Position of box, installation space and attachment points.

This task is typical in that it deals with the design of a mounting for a component which must remain at rest although several forces are applied to it.
A second task is concerned with the design of a mechanism for moving parts in a machine. It is taken from a diploma thesis of a student who had to model an existing piece of equipment for moving and cutting material which is used in the production line of a local company. In the task, part of the mechanism is to be designed where geometric measurements are given as depicted in Figure 2. There is a feed unit ("Vorschubeinheit") that moves the material forward for a certain distance, and there

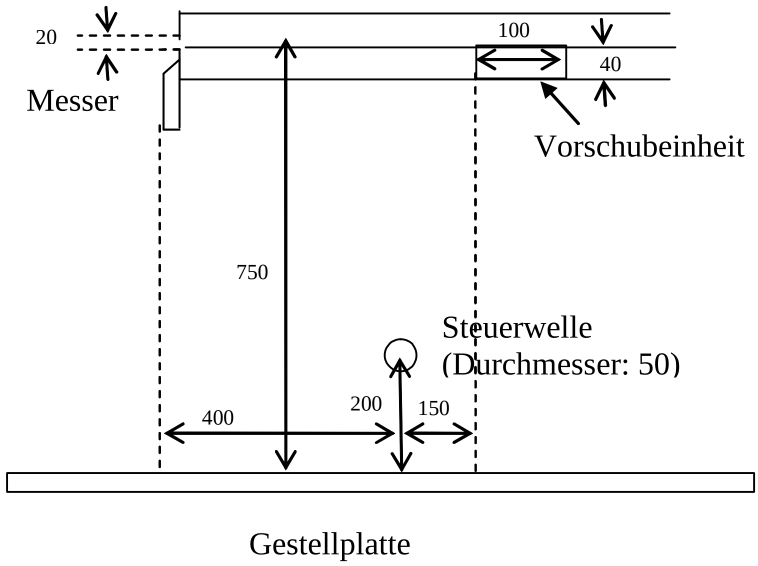

Figure 2: Surrounding parts and measurements for mechanism task.

is a knife unit ("Messer") cutting the material off. Both units are geared by cams rotating on a cam shaft ("Steuerwelle"). Since in reality, engineers always look at existing designs to get ideas and rarely work from scratch, we provided the students with

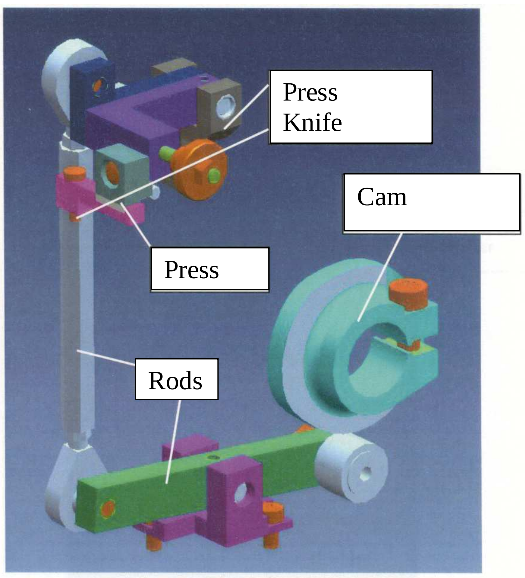

Figure 3: Similar mechanism.

a picture of a mechanism consisting of a cam and some rods and bearings so that they know in principle what the mechanism to be designed might roughly look like (Figure 3).

4. Preliminary results

So far, we have investigated how the students worked on the first task. Figure 4 below depicts the final constructions the students came up with. In order to achieve these results, students made heavy use of CAD programs and of a FEM program that was integrated into the CAD user interface. We briefly summarise the mathematical qualities that were exhibited during the usage of the programs. For CAD programs, we give a more detailed account in (Alpers, 2006).

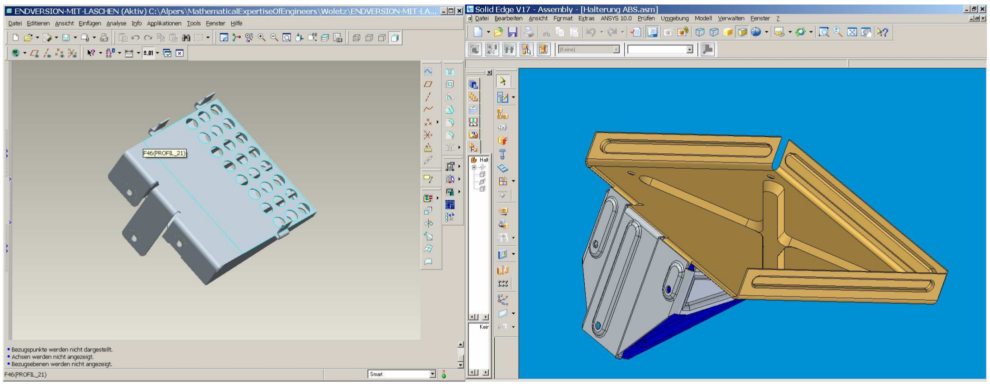

Figure 4: Constructions provided by students.

CAD programs embody a variety of geometric concepts and procedures (see, for example, Velichova, 2000, for an overview) but here we are only interested in those ones which are important for users (developers) to enable them to work effectively and efficiently with the program.

The students themselves could not identify much mathematical thinking within their work, but a closer look revealed that this originates from an identification of mathematics with symbols (algebra or differential calculus) which the students experience in their mathematical education. Although indeed, symbolic algebra and functions only show up in some advanced, more sophisticated areas of the CAD programs, the major part is also strongly influenced by mathematical concepts which can be roughly described as *operational synthetic geometry* for setting up constructions from basic blocks, and *relational geometry* for fixing the geometry and the position of a part by defining relations (distances, parallelism, orthogonality and so on). An example of the latter is given in figure 5 where the cross section of a cross crimping (depression of material to make a metal sheet stiffer) is shown. The students who made this design worked with distances in order to specify the desired relations (parallelism, symmetry).

The use of relations is also important for setting up constructions that can be easily modified and adapted. From this point of view the approach shown in Figure 5 is not very efficient since many distance measures have to be changed when, for example, the width of the crimping is to be altered.

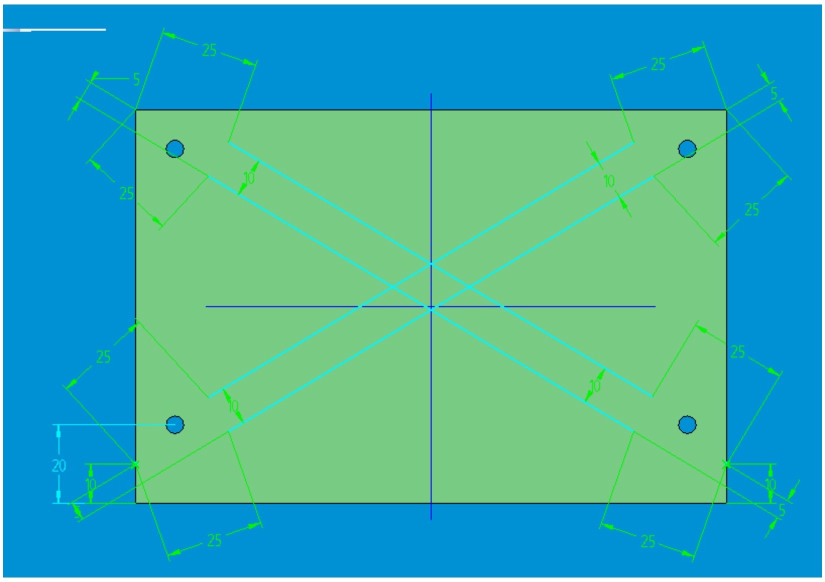

Figure 5: Part of sketch for cross-section of crimping.

Whereas in the CAD design activity, many mathematical concepts were displayed, in the computational FEM part the mathematics was almost completely hidden within the application. It seems to be a major design principle of the special "FEM for design engineers" version of the program to hide the mathematics away as far as possible. The user can transfer the construction directly from CAD to FEM and then has to provide information on forces, loads and bearings. Forces can be input as vectors, so the essential expertise seems to be the coordinate representation of vectors. Other important concepts in FEM (choice of element and functions, triangulation, etc.) are more or less hidden. When checking the results, the user is restricted to some plausibility rules. Since the constructions depicted in Figure 4 include crimpings and fins a rough computation using simplified models was not performed by the students simply because it is no longer possible, according to the application colleague involved. One plausibility rule the students used was to check the stress near the connection points (bearings) which should be higher. This way they found out about an erroneous use of the program since one of the bearings was not considered to be one by the program.

Rules are also used when answering the question of how to modify a design in order to fulfil the stress and stiffness requirements. Fins and/or crimpings are added accordingly, material is added or removed where it is not needed. The software enabled the students to check very quickly whether a change in the construction had the desired effect. Using an iterative procedure, an acceptable solution is achieved rather quickly, where the number of iterations depends on the experience of the user. It is still open for debate whether the reasonable use of the rules is to some extent dependent on prior computation in simple examples, as was hypothesised by Kent and Noss (2003).

5. Conclusions

When drawing conclusions from the results of the project, one has to be very careful. First of all, one has to keep in mind that one major goal of the mathematical education of engineers is to enable them to work with the models used in their application subjects. So, even if a model is not observed in practical daily work, it should be included in the mathematical education when it is used by an application colleague simply in order to guarantee a coherent overall education. Secondly, one has to consider the restrictions of our approach with respect to representativeness.

Therefore, we consider it important to integrate examples and tasks from the CAD area in the "normal" mathematics education. We think that set operations (union etc.) can be made much more meaningful for students when they have bodies in a CAD system as representations for sets (besides number sets or Venn diagrams). In geometry education, one can also deal with tasks where a sketch is given like the one in Figure 5 (but without relations or measurements) and students have to think about a complete set of relations to determine it (maybe even requiring some properties to be preserved when one quantity is modified). Such tasks can be quite challenging and initiate deeper thinking processes than are necessary for carrying out computational procedures.

As another positive side effect of this investigation, the author is better capable of understanding the geometrical thinking and reasoning of the students that comes from their preoccupation with CAD programs. The author can also make use of the knowledge acquired in this project for setting up meaningful mathematical application projects for mechanical engineering students (Alpers, 2002). The project is ongoing since a richer and more complete "picture" can only be obtained when several different tasks have been investigated.

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A New and Easy Access to Computer Algebra

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Since the early 90's many lecturers have experimented with computer algebra (CA) in mathematics courses. In several workshops experiences have been exchanged and it emerged that nearly all results were ambivalent: good students improve their performance when using CA, whereas students bad at mathematics become even worse. Here are some of the reasons:

- Many students do not know the English names of mathematical objects and operations (for example, denominator, numerator, curl);
- They are confronted with the syntax of a new language (for example, usage of parentheses, braces, brackets, comma, semicolon, difference between options and parameters);
- In some CA system they even have to learn abbreviations such as evalf, nops, op, subs;
- The structure of CA systems is quite abstract and turns out to be troublesome even for good students.

At the Computer Algebra Symposium Konstanz (CASK) 2003, Alpers, having experienced these same problems with his students using CA, presented maplets he had developed to address these issues. (A maplet is a Maple program creating windows containing such things as buttons and dialogue boxes). For each exercise there is a maplet giving a dialogue box with some buttons using German labels with the Maple commands hidden behind them. Students just have to find out in which order to press these buttons. Students using maplets need not learn the Maple language and syntax. For the lecturer it requires a huge amount of time to develop such a maplet and unfortunately it is only valid for one exercise. However, it turned out that weak students improved their performance by using these maplets.

The discussion following Alpers' talk showed the desire for a tool that meets the demands of students and lecturers as well:

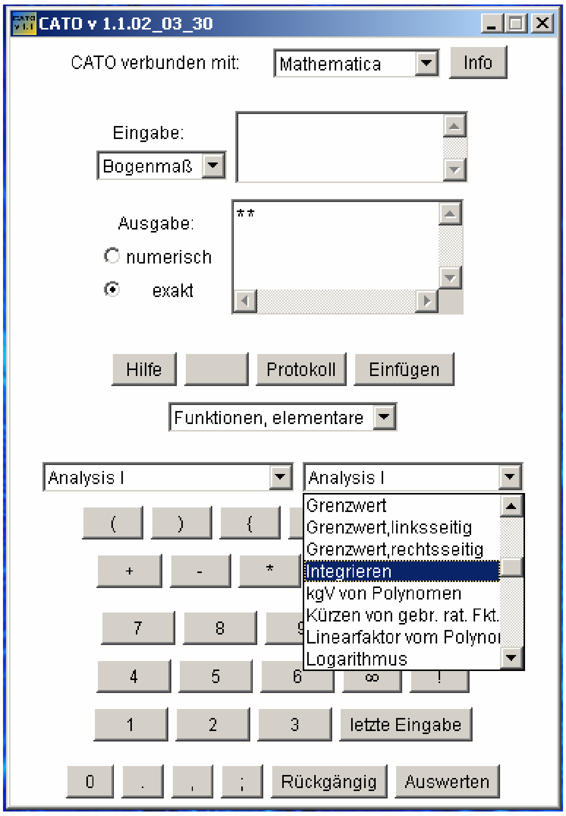
- The used language should be the student's mother tongue;
- It should be easy to use;
- There should be no need for the lecturer to produce a program for each exercise separately.

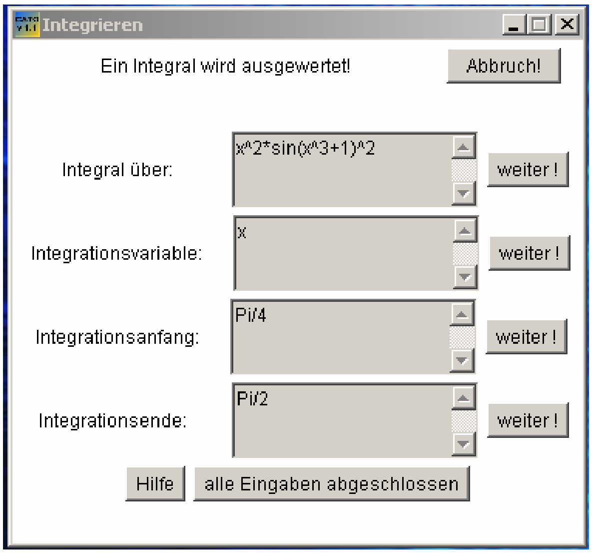
This discussion stimulated the development of a new CA interface CATO® which can be used with any CA-system having access to Java (at the moment Mathematica®, MuPAD® and Maple® Release 10). This interface looks like a pocket calculator and can be used quite intuitively.

For example, suppose you want to calculate an integral such as

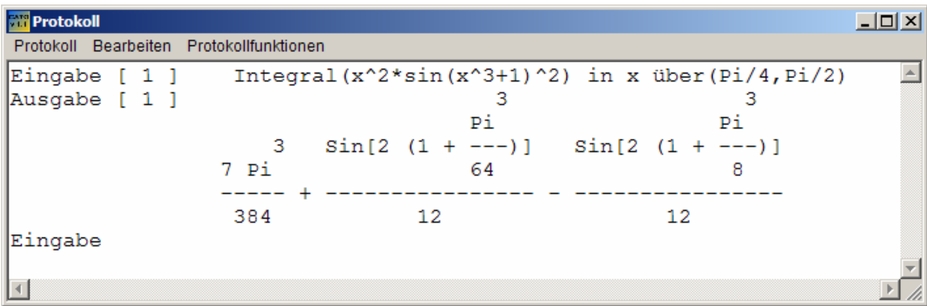
$$\int_{\pi/4}^{\pi/2} x^2 \cdot (\sin(x^3+1))^2\, dx\,.$$

You choose the package “Analysis I” (“Calculus I“) and in it the command “Integrieren“ (“Integrate“), then you just fill the gaps one after the other using the button “weiter!“ (“continue!“). If you do not know whether to choose “Integrieren” (“definite integral”) or “Stammfunktion” (“indefinite integral”) you can use the HTML help.





The result is quite voluminous, so it is helpful to look at it in the Protokoll (logfile) shown below.



For 2 years I have experimented with this new tool. Students can choose whether to use the normal CAS interface or CATO® during lectures and tests. Most of them decide in favour of CATO®, and since this allows them to concentrate on the heart of the matter, namely the mathematics and its applications to engineering, they improve their performance substantially. The failure rate has been diminished from about 25% to 6 - 7%.

Developments in Mathematics Support in the United Kingdom

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Abstract

In recent years, concerns have been expressed in several countries in Europe about the mathematical competences of incoming undergraduate students, particularly those on courses such as engineering where mathematics, although a necessary component, is not the main subject of study. In the United Kingdom, universities have developed a number of strategies to help address the difficulties that this change in entry competences brings about. In this paper, we will describe a number of such developments over the last 15 years in which Coventry and Loughborough Universities have played a leading role.

1. Introduction

Towards the end of the 20th century, a number of reports (for example [1-4]) were published in United Kingdom which indicated that the mathematical competence of students on entry to university had changed significantly. These reports drew on data such as that of Hunt and Lawson [5], Lawson [6] and Todd [7]. These data recorded the performance of new undergraduates on diagnostic tests taken during their first week at university. The tests were kept the same year on year but the number of marks scored by students declined consistently throughout the 1990s. For example, data from Lawson, quoted in [4], shows that students entering Coventry University in 1999 with an A level mathematics grade B (the second highest grade) scored on average the same mark in the diagnostic test as students entering in 1991 with A level mathematics grade N (a fail grade).

This is not solely a British phenomenon. A decline in the mathematical competence of incoming undergraduate students has also been reported from several other European countries [for example, 8-10] and also internationally [11]. Several factors have been suggested as contributing to this decline. These include changes in the way mathematics is both taught and examined in pre-university education; an increase, in many countries, of the proportion of the age cohort entering university; a wider range of subjects on offer in both pre-university and university education leading to lower popularity of science and engineering courses.

Despite the fact that students are entering university with less well-developed mathematical skills than their peers a decade earlier, there is pressure on universities to maintain output standards. One way to achieve this might be to give students additional teaching time at university to enhance their basic mathematical skills.

However, this is rarely an option for two main reasons. Firstly, in many countries there has been a drive to increase 'efficiency' in universities; this often means reducing the amount of timetabled teaching time that students receive. Secondly, in subjects like engineering there has been pressure to extend the curriculum to introduce more coverage of topics like business, management and language skills. As a consequence, mathematics has often received a smaller share of a smaller timetabled teaching allocation.

In the face of these twin pressure creative solutions have been needed in order for mathematics to be able to help students fulfil their potential in mathematics. In the remainder of this paper we will outline some of the approaches that have been introduced in United Kingdom that have provided benefit to some students and also some very recent innovations which it is hoped will offer support to a wider range of students.

2. Mathematics Support Centres

Many British universities now offer some form of mathematics support provision. A survey carried out in 2000 [12] indicated that almost 50% of universities did so and since then several universities have begun to provide this kind of support. By 'mathematics support' what is meant is the provision of additional learning opportunities for students in addition to their regular scheduled lectures, tutorials and problem classes. In its most basic form this could just be a few hours a week when a member of staff is available to help any student who is having difficulties with the mathematics within their programme of study. More developed forms of this support provide a range of learning resources in a dedicated facility to enable students to undertake systematic programmes of work to improve their mathematical skills.

Two of the leading support centres in Britain are the Mathematics Support Centre [13] at Coventry University and the Mathematics Learning Support Centre [14] at Loughborough University. Both these Centres provide dedicated accommodation which is staffed for approximately 30 hours each week. During these hours, students can 'drop-in' (i.e. come without appointment) to discuss any mathematical or statistical topic on a one-to-one basis with a member of staff. An extensive range of paper-based resources is freely available for students to use either within the centre or to take away and use elsewhere. The Centres have their own web-sites [15, 16] making available further resources, such as on-line practice examples, and computer-based learning resources are also available for use within the Centres.

Initial diagnostic testing is an important element of mathematics support at many universities [17]. The purpose of this is to help identify individual student's mathematical strengths and weaknesses. Based on their performance in the test, students receive a personal diagnosis of mathematical topics where it would be beneficial for them to develop their skills. They are then encouraged to make use of the resources (human, paper and electronic) of the mathematics support centre to improve in these areas.

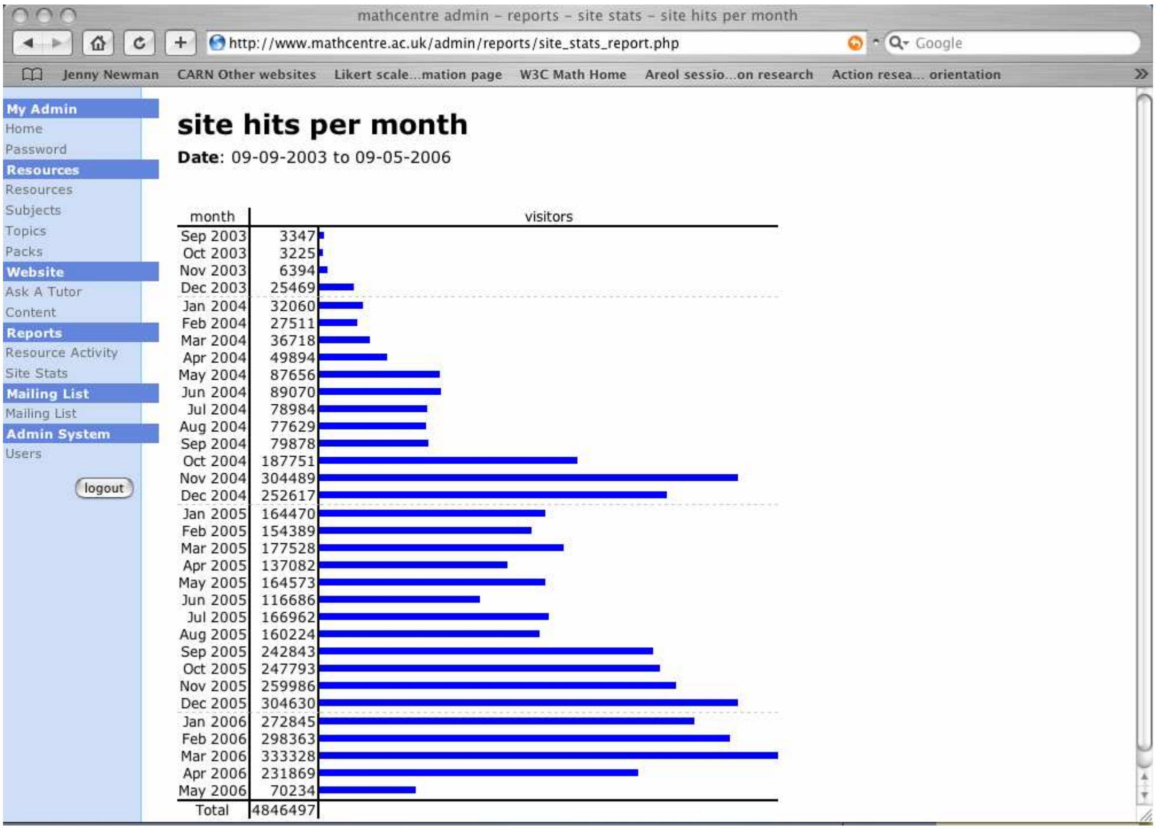

Figure 1: Number of hits per month on the mathcentre web-site

In order to make a reasonable level of support available to students in all universities, a virtual mathematics support centre was created in 2002. Funded by the Learning and Teaching Support Centre, mathcentre (www.mathcentre.ac.uk) was designed to provide easy access to a structured array of resources focused primarily on supporting the transition from school to university mathematics, particularly for students not taking specialist mathematics degrees. There are two ways of accessing mathcentre – as a student or as a member of staff. The main difference between the two forms of access is the range of materials made available. Staff can access everything available to students, but in addition there are links to teaching resources (such as the MathsTEAM booklets [17-19]) and bundles of handouts (these are available to students individually but not in bundles).

Figure 1 indicates the number of hits on the mathcentre web-site on a month by month basis. The peak month to date was March 2006 when there were almost one third of a million hits on the site (i.e. in excess of 10,000 hits per day). Throughout the current academic year (i.e. since Oct 2005) there have been around a quarter of a million hits per month (April dropped slightly below this level because of the Easter vacation). The volume of traffic on the site indicates that many students regard this as a very valuable resource. In addition, the site regularly receives unsolicited emails from users praising the quality of the resources made available on the site.

3. Video-based Resources

When evaluation of mathematics support centres is carried out, students invariably rate the interaction with tutors as the most valuable resource provided by the centres. In order to add a human element to the electronic resources provided by the mathcentre site, a supplementary project, mathtutor, has been undertaken. This project, resourced by the Fund for the Development of Teaching and Learning [20], was undertaken by a consortium consisting of Coventry University, Educational Broadcasting Trust, Leeds University and Loughborough University. It set out to provide, on DVD-ROM, a set of learning resources based around a video tutorial delivered by a "master teacher". The resources cover material at the school university interface and for each topic there is a video tutorial supported by an initial diagnostic test, summary text which follows the exposition in the video tutorial and interactive exercises. Some topics also have extension material which cover additional material related to the topic – this may give an interesting application or go into greater depth.

Sample screens from the unit on the chain rule for differentiation are shown in Figures 2 and 3. Figure 2 shows the entry page once the topic of the chain rule has

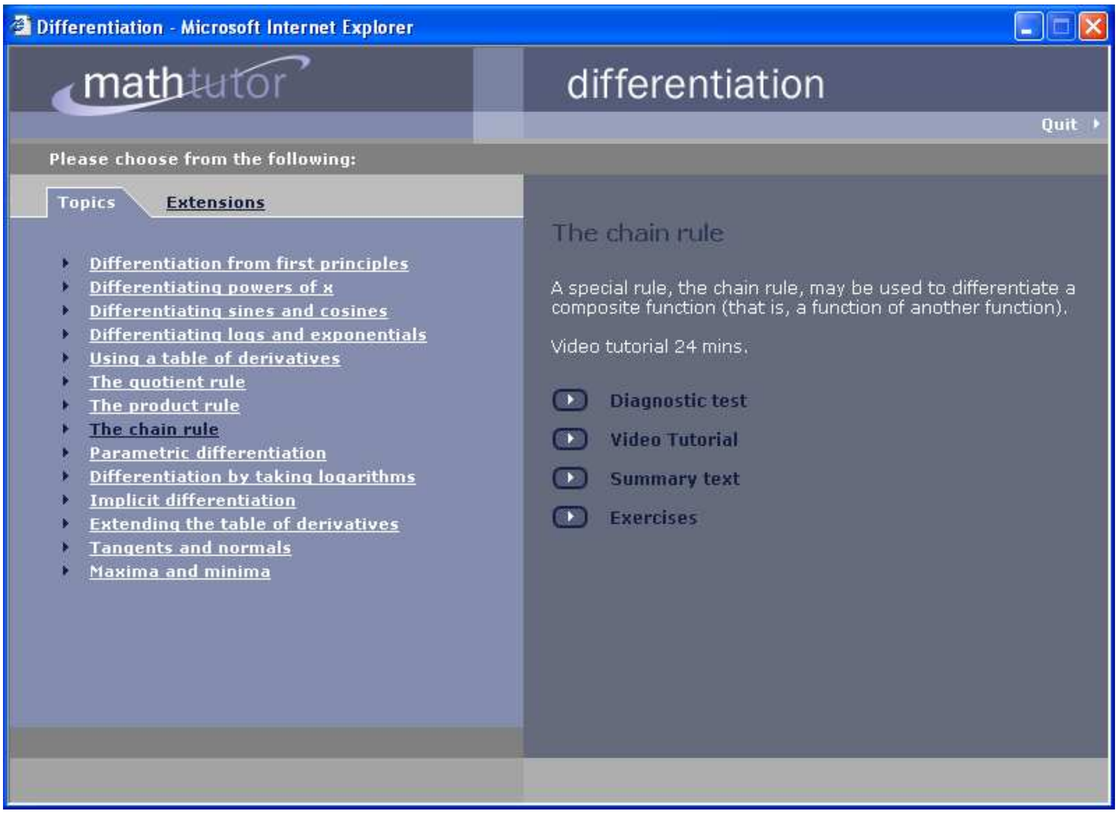

Figure 2: Menu Screen from the Differentiation Disk

been selected on the differentiation disk. The four main resources are presented in a menu. Students can choose to take the diagnostic test in order to assess how familiar they are with the topic. As a result of taking the test students may decide that they do not need to view the video tutorial or that they need to watch some or all of it. The video tutorials have multiple entry points (shown on the left half of the screen in Figure 3). These allow students to begin viewing a tutorial at an appropriate point without having to sit through material with which they are already familiar. Students can download and print the summary text and have this at hand whilst watching the tutorial. At any point, they can pause the tutorial and try some of the interactive exercises.

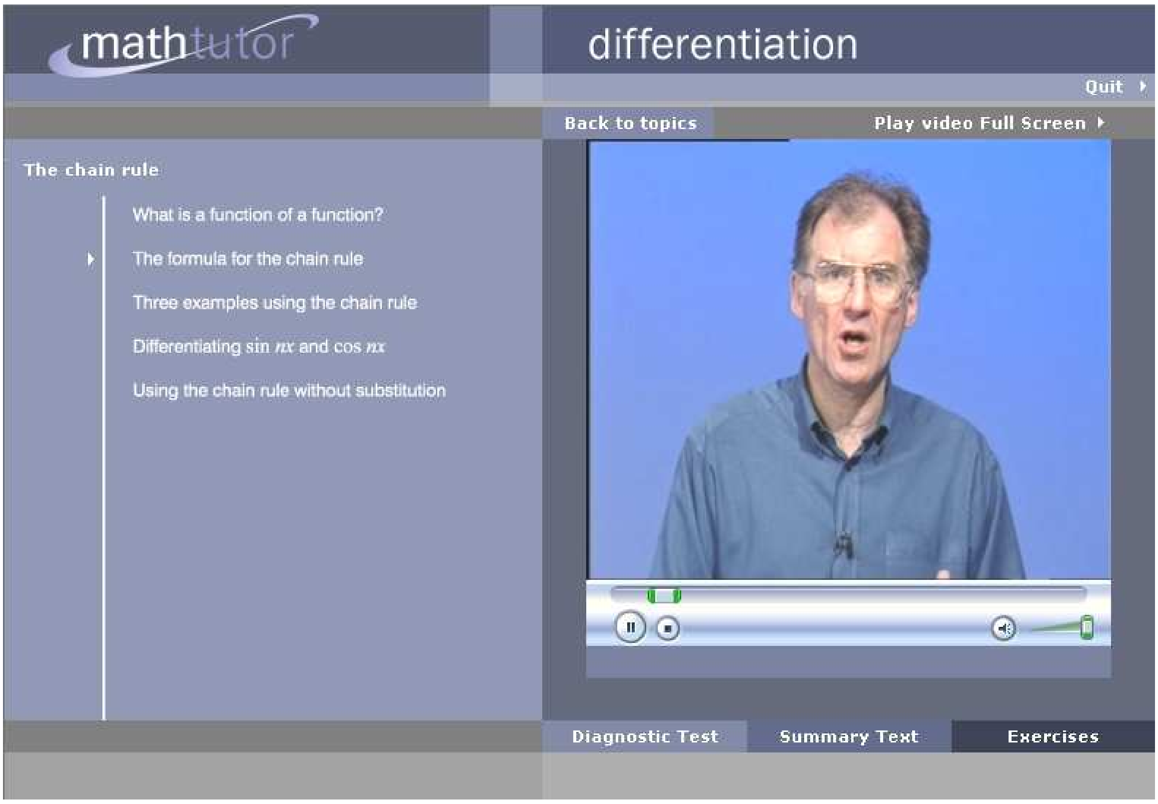

Figure 3: Video Tutorial Screen for the Chain Rule

In total, there are 85 video tutorials on seven DVD-ROMs covering:

- Arithmetic
- Algebra
- Functions & Graphs and Sequences & Series
- Geometry and Vectors
- Trigonometry
- Differentiation
- Integration

At the commencement of the project in 2002 DVD-ROMs were regarded as the only viable delivery mechanism for the material being developed. However, advances in technology during the lifetime of the project have meant that the tutorials could be delivered over the internet. Initially the video tutorials alone were made available on the mathcentre web-site, however recently the full functionality of the disks has been released at a new web-site www.mathtutor.ac.uk.

4. Centre for Excellence in Teaching and Learning (CETL)

In 2004, the Higher Education Funding Council for England (HEFCE) announced the Centres for Excellence in Teaching and Learning (CETL) initiative [21]. This had two main aims: to reward excellent teaching practice, and to further invest in that practice so that CETL funding delivers substantial benefits to students, teachers and institutions. A joint bid from the Mathematics Education Centre (which incorporates the Mathematics Learning Support Centre) at Loughborough University and the Mathematics Support Centre at Coventry University was successful and a new centre, SIGMA [22] was recognised as a Centre for Excellence in University-wide Mathematics and Statistics support.

SIGMAs aims are that students will:

- Have a better understanding of the importance of mathematics and statistics to their mainstream discipline;
- Develop more effective learning approaches to mathematics and statistics;
- Be proficient at using mathematics and statistics within their mainstream discipline;
- Succeed in greater numbers.

SIGMA began operating in the summer of 2005. Its key activities to date have included:

- Additional support for less well-prepared students through small group and/or additional teaching;
- Provision of statistics support (short course and one-to-one consultation) for students undertaking project work (undergraduate and postgraduate);
- Support for students with disabilities and learning difficulties such as dyslexia;
- Investigation of the use of new and emerging technologies in mathematics support;
- Pedagogic research.

As SIGMA has been operational for less than a year it is too soon to be able to draw definite conclusions about the effectiveness of its activities. However, early evaluation of some activities indicate a positive impact on student performance [23].

5. Mobile Technology

One area that SIGMA has actively investigated is the use of mobile technology. In October 2005, Apple Computers released the video iPod [24]. The iPod has for some time been the market leader in mobile music technology. This latest iPod has a high resoution screen and can display videos at extremely high quality.

Figures 4a and b: Video iPod menu system and displaying mathematics

The video materials from mathtutor have been converted so that they run on a video iPod. Generally the video tutorials are longer than is ideal for this medium, however the animations are much shorter. A range of 15 animations can be downloaded in video iPod format from http://www.mathtutor.ac.uk/ipod.shtml. Some of the complete tutorials are available for download from the mathcentre site by searching for iPod videos. Work is underway to extract short segments from the complete videos, such as individual worked examples and these will be made available shortly.

Figures 4a and 4b show the video iPod menu system and the screen with a video running. The resolution of the screen enables the written mathematics, although small, to be clearly legible.

At present, not many students have video iPods. However, it is anticipated that, as happened with MP3 players, other manufacturers will soon enter the market with similar products at cheaper prices. Our intention is to develop a bank of video-based resources that students will be able to download to view on their mobile device. This development is at an early stage and it is not yet possible to predict with certainty how effective this method of student support will be.

6. Conclusions

The range of mathematical competences of students entering universities is growing ever wider. The best students are still very well prepared, but the less well-prepared have significant gaps in their knowledge and competences. In order for the majority of these students to succeed it is necessary to have a range of support provision available. In Britain a number of measures have been introduced to assist such students. These measures are both real (physical support centres staffed by academics) and virtual (on-line resources including video tutorials and interactive exercises). As technology develops so new opportunities arise for providing support to students and it is important that the effectiveness of these opportunities is explored and evaluated.

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Coming to terms with change: mathematics for engineering undergraduates

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Abstract

Looking back over thirty-six years of teaching mathematics to engineering undergraduates, the author examines the changes that have taken place in the undergraduate population and how the teaching of mathematics has responded.

Tension cracks have opened up for university teachers of mathematics to engineering undergraduates as a result of a lowering of the mathematical preparedness of undergraduates and the continued high expectation of their mathematical fluency by other engineering academic staff. Changes in technology have brought their own problems: how should we adjust to their existence and incorporate them in (or ignore them from) our teaching? The whole nature of the syllabus content and the modes of its delivery need close scrutiny.

What is the purpose in teaching mathematics to engineers? What skills and competencies do we, and should we, expect them to possess by the end of their mathematics courses? Given the decreasing mathematical fluency of new undergraduates, how far down the path of easing our expectations and demands should we go? This paper considers these questions and suggests some answers.

1. Introduction

Most branches of engineering rely on mathematics as a language of description and analysis. "Engineering is based on the inter-dependence of scientific and mathematical understanding with problem-solving and inventive skills." "Fluency and confidence in the knowledge and use of mathematics comes with repetition and practice but deeper understanding comes with the experience of transferring concepts and principles to new contexts and applications." [**1**]. During recent years engineering departments have had to deal with a growing number of educational issues relating to mathematics. Unlike many other disciplines engineering courses have had problems attracting a sufficiently high proportion of the more able entrants [**2**], [**3**]. This has resulted in an increasingly diverse range of levels and types of qualifications at entry into engineering courses. There is also growing evidence that many entrants lack basic mathematical skills, a situation which has become well documented during recent years [**4**], [**5**], [**6**].

Universities have taken a range of steps in an attempt to address this situation. There have been curriculum changes so that more 'revision' material is included in the first year, provision has been made for extra support units to be studied alongside the traditional syllabus, some departments have introduced bridging units to be studied intensively at the start of the course, and Mathematics Support Centres have been established [**7**]. Such support might be seen as applying sticking plaster rather than providing students with the coherent knowledge base needed to underpin an engineering course. The Royal Society report *Engineers – The Supply Side* [**1**] argued in favour of the core curriculum being broadened and made more flexible.

To plan for the future it is necessary to understand how we have reached the present. The experience of the United Kingdom may not be typical but it is offered as a warning, working on the principle that in the Northern hemisphere bad influences in education spread from west to east.

In the immediate post-war era Britain experienced a new industrial awakening. It was recognised in the late nineteen-fifties that engineering courses were not sufficiently sophisticated; for example, the traditional teaching of strength of materials was inadequate for the speedy and safe design of pressure vessels needed in the power industry; as a second example, the study of high speed flight demanded a familiarity with three-dimensional compressible flow. Engineering departments asked their mathematics colleagues to up-grade their courses, and topics such as Laplace transforms, functions of a complex variable, and matrix and vector analysis became standard.

After the initial up-grading of mathematics in engineering courses things stabilised for a while, but even twenty-eight years ago there were rumblings of discontent. The author warned [**8**] that unless mathematics departments took more care to liaise with their engineering colleagues, and were prepared to modify the mathematics modules that they offered, then they faced the prospect of losing that teaching to the engineers themselves.

Engineering courses are proving less and less popular with candidates for university entrance. The lure of what appear to be more interesting, and (dare one say) less demanding, courses puts pressure on engineering departments to lower entrance requirements in order to fill places. In the United Kingdom the popular scapegoat for the unattractiveness to applicants of engineering courses is mathematics.

2. Is mathematics swimming against the tide?

In the mid nineteen-sixties the secondary level mathematics syllabuses underwent a sea-change, moving to a more 'modern' approach. Euclidean geometry all but disappeared, set theoretic concepts came in and matrices were used to code and decode messages. Coordinate geometry was demoted, to be replaced by elementary work on differential equations, vector algebra and matrix algebra. It became clear that the mathematics being taught was too theoretical to be treated in a suitable fashion and rested on foundations that were too shaky. There was a subsequent partial retreat to more traditional waters, but the damage had been done.

The claim by successive governments was that the ‘gold standard’ of Advanced (A) level examinations, taken in the final two years of secondary education had to be, and was being, maintained. In 2000 a new qualification was introduced in the penultimate year – the Advanced Subsidiary (AS) level. This was alleged to be ‘half an A level’, but whether that meant in content or in depth is not clear. It was decided that, in an attempt to broaden the curriculum, four AS levels would be taken. This would be followed in the last year of secondary education by one of a variety of possible options, including three of the subjects being studied to A2 level in order to complete the old A level, or two at A2 and one new subject at AS.

The first year of operation of the AS in mathematics was a disaster, with a 29% failure rate. Probable causes include that fact the AS syllabus was ‘beefed up’ in response to requests from Higher Education when other subjects, taking cognisance of the fact that four subjects were being examined as opposed to three previously, moved in the opposite direction.

As fewer applications for engineering courses are received, so the need to dig deeper to fill places increases. This inevitably means taking candidates with lower grades than would be desirable; often the grade relaxation takes place in mathematics. The justification offered is that by providing a crash course in mathematics at the beginning of their studies, or even in the weeks before, the students concerned can catch up. Really?

It should scarcely be necessary to remark that today’s students are not the students of yesterday, although it sometimes seems that engineering colleagues often forget this, at least as regards the mathematics aspect of the course. In the last decade or so, a number of factors have conspired to make the task of those who lecture mathematics to engineering undergraduates more difficult. The main problems facing many students are: a lack of basic skills in number and algebra, engineering modules which assume knowledge and skills which some students do not have, and their inability to handle multi-stage problems. The first of these impedes the understanding of those topics which assume familiarity with these areas of mathematics. The skills and knowledge referred to in the third cannot be acquired easily or quickly.

The level of mathematical achievement of those with apparently comparable Advanced level qualifications appears to have declined, so that even when the requirements on entry grade levels are being maintained there is a problem. Just how bad is the situation? Mathematics lecturers, sometimes rightly so, have been accused of teaching at a far too theoretical level, but we are talking here about lack of fluency in really basic mathematics.

Engineering colleagues bemoan the fact that their students no longer seem to possess a feel for order of magnitude, are unable to carry out simple algebraic manipulation, know very little elementary geometry, are bewildered by relatively simple trigonometric expressions and are able to attempt only the simplest aspects of differentiation. So it is, and one might add that when confronted by a mathematical problem the students will respond with 'can't do it' if the solution requires more than the direct application of a method covered in lectures. The catalogue does not end there, unfortunately; there appears to be very little work ethic among new students, which, for a demanding subject like engineering, could sound the death knell for the degree programmes of today. For mathematics at whatever level the only sure recipe for success is practice and more practice.

Some of the changes in undergraduate attitudes are societal and have affected also the teaching and learning of subjects other than mathematics; have those subjects responded more sympathetically and more effectively? Probably. Some changes have had more effect on mathematics and mathematics-related subjects: does this imply a more substantial response is required in these areas? Probably.

As departments face an increasingly diverse range of levels and types of qualification, there is evidence of new systems being introduced, changes to the course structures and the introduction of different methods of teaching **[9]**.

3. Is the team pulling together?

Mustoe's warning [**8**] has proved all too prophetic; many mathematics departments have lost teaching to the engineers, indeed many mathematics departments have closed as a result of the loss of the attendant income. In the early stages this transfer of teaching was largely motivated by educational considerations; today, financial considerations have entered the equation.

Even when the mathematics departments have retained the teaching there is pressure to reduce the hours for mathematics. There has been a move to 'broaden the curriculum' by including topics such as management, economics and the like. In order to find the room for these topics something has to make way. All too often mathematics is at the top of the list. The defence is sometimes put up that engineers 'do not need so much mathematics, it can all be done on the computer'. Furthermore, the task of providing a coherent mathematics education seems to be disregarded by those who advocate a project-based approach to engineering education, or worse, a 'just-in-time' philosophy.

In 2004 the Engineering Council of the UK revised its Standards and Routes to Registration (SARTOR3) to a Standard for Professional Engineering Competence (UK-spec) **[10]**. Some key aspects of SARTOR3 were not changed: registration is still based upon competence and commitment, there are three registration categories – Chartered Engineer (CEng), Incorporated Engineer (IEng) and Engineering Technician (EngTech), and there are academic benchmark levels.

The Council state that Chartered Engineers are characterised by "their ability to develop appropriate solutions to engineering problems, using new or existing technologies, through creativity, innovation and change. They might develop and apply new technologies, promote advanced designs and design methods, introduce new and more efficient production techniques…are variously engaged in technical and commercial leadership and possess effective interpersonal skills". Incorporated Engineers are characterised by "their ability to act as exponents of today's technology through creativity and innovation…they maintain and manage applications of current and developing technology…are variously engaged in technical and commercial management and possess effective interpersonal skills".

In addition to specifying general learning outcomes, UK-spec has a set of specific learning outcomes, the first two of which are "Underpinning science and mathematics, and associated engineering disciplines, as defined by the relevant engineering institution" and Engineering Analysis. It specifically refers to mathematics by requiring registrants to have "Knowledge and understanding of mathematical principles necessary to underpin their education in their engineering discipline and to enable them to apply mathematical methods, tools and notations proficiently in the analysis and solution of engineering problems". The requirements for Engineering Analysis are "Understanding of key engineering principles and the ability to apply them to analyse key engineering processes", "Ability to identify, classify and describe performance of systems and components through the use of analytical methods and modelling techniques", "Ability to apply quantitative methods and computer software relevant to their engineering discipline, in order to solve engineering problems" and "Understanding of ability to apply a systems approach to engineering problems".

For accredited MEng courses **"**A comprehensive knowledge and understanding of mathematical and computer models relevant to the engineering discipline, and an appreciation of their limitations" is expected and in reference to the Engineering Analysis aspect "Ability to use fundamental knowledge to investigate new and emerging technologies", "Ability to apply mathematical and computer-based models for solving problems in engineering, and the ability to assess the limitations of particular cases" and "Ability to extract data pertinent to an unfamiliar problem, and apply in its solution using computer based engineering tools when appropriate" are all required.

For accredited IEng courses **"**Knowledge and understanding of mathematics necessary to support application of key engineering principles" is expected. Furthermore, in reference to the Engineering Analysis aspect "Ability to monitor, interpret and apply the results of analysis and modelling", "Ability to apply quantitative methods relevant to the discipline", "Ability to use the results of analysis to solve engineering problems, apply technology and implement engineering processes" and "Ability to apply a systems approach through know-how of relevant technologies" are all required.

The tension cracks that have opened up for university teachers of mathematics to engineering undergraduates are the result of a diminution in the mathematical preparedness of undergraduates and the continued high expectation of their mathematical fluency by other engineering academic staff. These cracks are likely to widen in the near future. We therefore need both a short-term and a medium-term plan of action.

4. The effect of changing technology

More than thirty years ago, in a response to the increasing use of computers to solve engineering problems, mathematics departments introduced modules in numerical methods. The computing power available to students included electro-mechanical calculators; it was so far removed from today's facilities that it seems almost Neolithic.

At Loughborough in 1970 the author piloted a course in which numerical methods were taught alongside companion analytical methods where possible [**11**]. For example, when introducing first year students to differential equations a problem relating to their particular engineering discipline would be used to set the scene; then simple analytical and numerical methods of solution of the resulting differential equation would be presented, allowing a comparison between them to be made. This problem-centred approach seemed a more natural way of presenting the material, whilst retaining the integrity of the mathematics.

During the last thirty years, learning technology has become more widely available. In relation to the engineering mathematics curriculum, this has enabled new approaches to teaching and learning. In addition, sophisticated mathematical software is now commonly available which routinely allows analysis of problems of such size and complexity that only a few years ago would have been regarded as a research activity **[2]**.

There are definite signs that the involvement of technology in the teaching of undergraduate engineering mathematics is beginning to gain momentum **[9]**. For example, the graphics calculator has become an integral part of some courses; widely available software such as spreadsheets is used as a powerful tool for data manipulation; 'specialist software' is also playing an important role in teaching mathematics to engineering students.

Additionally, computer-programming skills can become a central part of teaching engineering students. In some courses the emphasis is very much on practicalities. How do numerical methods work? What can go wrong? How do I write a programme for this method? How can I adapt someone else's programme? Only what we see as the very minimum necessary amount of theory is developed and issues such as numerical stability are mainly addressed by experiment **[12]**.

Mathematical technology is therefore becoming an integral part of teaching in some institutions, while others still base their contextual teaching on the pen and paper approach.

These changes in technology have brought their own problems. Although often misused, the pocket calculator, with ever-increasing sophisticated facilities, is here to stay; it is now accepted as part of a student's standard equipment. But the fact that it is often misused sends alarm bells ringing as ever more powerful computing facilities become readily available: just how should we adjust to their existence and incorporate them in (or ignore them from) our teaching? And it is not simply the fact that the more powerful the computational aid the more damaging its misuse can be. The whole nature of the syllabus content and the modes of its delivery must come under close scrutiny.

5. Where do we go from here?

A number of departments have developed innovative teaching approaches across various levels of study, thus giving context to mathematics within programmes of study and broadening the learning capabilities of the students **[9]**. For other departments, context is achieved by directing the mathematics teaching towards a specific level of student knowledge. Courses are taught using relevant examples, in an order related to the degree programme rather than that found in traditional mathematics courses.

Fundamental questions need to be asked. What is the purpose in teaching mathematics to engineers? What skills and competencies do we, and should we, expect them to possess by the end of their mathematics courses? (In this context "we" means both mathematics and other engineering academic staff.) Given the decreasing mathematical fluency of new undergraduates, how far down the path of easing our expectations and demands should we go? Have we already passed the point of no return?

The dilemma facing the mathematics lecturer is whether to retreat and, if so, how far to retreat. The lesson of Danegeld in English history is pertinent. In order to buy off the Danish invaders the Saxons offered them parts of Eastern England and an annual tribute, known as Danegeld, provided that they made no further incursions westwards. The Danes took the money and still made further incursions. Every time that the mathematics syllabus is watered down or the time allocated for mathematics is reduced it is a one-way step. For how much longer can we continue along this path?

The SEFI Mathematics Working Group produced a revised version of its Core Curriculum in mathematics for the European Engineer [**2**] and anything less than this would be unwelcome. We must work with our engineering colleagues in order to defend that core. It is pertinent to ask whether, even so soon after its appearance, the Core Curriculum needs revision.

6. Conclusions

Academics face the task of ensuring that mathematical understanding does not become a scarce ability amongst graduate engineers. As institutions face the prospect of entrants entering engineering courses with a widening range of abilities, the task is ongoing and requires academics to continue innovating, changing and adapting. On the whole, it is through these innovative teaching methods and technology that students are becoming actively involved in both the learning process and in the processes of the discipline.

We must be active in pursuing our aims or else we shall see mathematics, in the words of the former British Prime Minister Harold Macmillan, slide gracefully beneath the waves like a great liner.

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Computer Exercises in Teaching Mathematics

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With the transition to the three-level study system at university the contents of the mathematical curriculum have been compressed in time. Simultaneously, there has been a radical reduction in the number of lessons, especially exercise classes. This brings a series of problems and the necessity to change the standard forms and methods of working. We have introduced so-called "Computer exercises" into regular teaching. Here we put significant emphasis on the individual responsibility of the student, not only during teaching but also when undertaking homework.

Computer exercises take place once every fourteen days, in the specialized Information Technology room which holds 24 students. In this room, in addition to computers with the required software, there are data projection and overhead projection facilities. Students are introduced to the mathematical programme MATLAB, for numeric and symbolic computation, for high-level programming and for advanced graphics and visualization. Because students use MATLAB in their subsequent studies, especially in technical subjects, we concentrate on these topics:

– Elementary mathematical functions
– Polynomials, Partial-fraction expansion, Matrix analysis
– Graph of a real function
– Scalar nonlinear zero finding, Scalar bounded nonlinear function minimization, Differences
– MATLAB advanced graphics and visualization
– Analytical geometry, Vector cross product, Vector dot product
– Conic Section, Quadratic Surface

During the semester students have to complete four so called partial tasks, investigate the graph of a real function and undertake a one semester project. A student can receive up to one point for each partial task, but he must achieve at least 0.5 point, in order for it to be accepted. For the graph of a real function he can receive two points: one point is necessary for it to be accepted. For the semester project he can receive 2.5 points: two points is necessary for it to be accepted. For activity during the exercises he can get one point and for attendance 0.5 point. Altogether he can receive ten points from the Computer exercises, which contributes to his grading in the examination.

Because the subject of Mathematics has too few lessons, we must choose a practical approach to teaching. At first students are introduced to a practical problem, then explanations of the problem are usually made and an example problem is solved. The main activity is independent and individual work by the students, who solve analogous problems with modified tasks. They can ask the teacher questions. The selected problems are used to show students the theoretical knowledge that is needed to solve the problem correctly. In addition, the importance of monitoring any MATLAB error messages is emphasized. We show the students how to elegantly and quickly find, using MATLAB, the solution of a system of equations which has a unique solution. Then we show systems which have infinitely many solutions and no solutions. Similarly, we illustrate the graph of a real function, finding zeros, searching for extrema and so on. All students have to do homework after each lesson for the next lesson in fourteen days time. This homework is individually assigned for each student.

The lessons have a very fast pace and students do not have time to make a complete set of notes. We have produced an "Accompanying text to the Computer Exercises from Mathematics I", which is not only a manual for the mathematical programme MATLAB, but also includes a large number of examples on the utilization of the selected commands and functions. This text is made available on the internet together with on-line versions of selected exercises and teaching texts which help students to solve problems.

The first partial task is a word problem from secondary school from different thematic parts and from different practical course; each student has a totally different problem. The objective for students is to find a mathematical method for solving the problem, to implement this method to find the solution, to write the solution on paper and to interpret the solution in the context of the original problem.

The second partial task deals with systems of equations and again each student has a completely different word problem. The objective is similar: to construct the appropriate system of equations, to solve it and to write the solution and its interpretation on paper.

The next homework is to produce a graph of a real function. Again each student has a totally different function and the objective is to use differential calculus according to fifteen predetermined points to complete a graph of a real function and to write a so-called m-file, which draws the graph of this real function including axes, asymptotes, tangents and so on. The completed document has to include all calculations, the m-file and a graph from MATLAB.

The third partial task is a problem from analytical geometry. Each student has a different problem which usually requires the calculation of the volume, surface area and height of some angular body.

The fourth partial problem is a word problem about the extrema of a function. Again each student has a different word problem from which he has to determine the function, find the extrema through calculation and then write up the correct answer.

All these problems are organized vertically and horizontally. This means that every set of problems covers different sections of the syllabus and contains problems of different levels of difficulty. Each student is given a group of five problems of different difficulty and nobody has only difficult or only easy or middle problems. A word document was constructed for each set of problems; this was named and consequently sent to the appropriate e-mail address.

In the semester work, the student has to show that he has learnt the required knowledge and skills not only in mathematics, but also in the application of MATLAB and computer technology. This is why the semester work is only available electronically either from the Faculty Intranet (in a directory in which students have only read access), or from the Internet. Students have to work out and hand in this work in electronic form, using the editor Word. The final document has to satisfy all the criteria given in advance and has to be saved on the Faculty Intranet in a directory in which students have only the right to create and scan files. This means that neither the author nor any other student can open, delete or read the file. The semester work contains five basic word problems, which are formulated so as to seem identical for all students. However, each student has a different variation of the problem. For example in the fifth problem students do not have only different numbers in the equation of their conic sections, but some of them analyse parabola and hyperbola, others ellipse and hyperbola and so on and moreover the conic axes are parallel to either *x* or *y*-axes.

Attention is focused on such problems where non-trivial and meaningful use of computers is required. For example, in the first problem, a polynomial of the fifth degree is in the denominator, and students are not able to solve it. They have to find the roots of this polynomial with the help of MATLAB and then make the partial-fraction expansion, for which they need to know the relevant theory. In the third problem they have to draw the relevant area in order to be able to formulate the integral, which will give the solution of their problem. Ignorance of the relevant theory and mere mechanical computation of the integral generally leads to an incorrect result. Similarity we can get incorrect graphical illustration of surfaces, conic sections and so on. Students work with different forms and different applications in the solution of particular problems and this way they learn to put the text, data or pictures from different applications into one document. A student should demonstrate in his semester work, that he knows how to use computer technology in the creation of a written document, that he is able to use the mathematical program MATLAB in the solution of mathematical problems, that he is able to graphically illustrate results obtained and to present them not only in the plane, but also in three-dimensional space and that he is ready to take responsibility for the results obtained.

The computer exercises should contribute both to better independence of students and to increasing interest in mathematics. A student should demonstrate that he is be able to apply the knowledge he has learnt in the solution of unseen problems, that he understands the connection of mathematics with other technical subjects, that he is able to solve problems and analyse the results. The future will show if this approach is successful and what results the students will achieve from using these carefully conceived computer exercises. So far it seems that the study of mathematics is very difficult for students. This is not only because of the necessity to master a large curriculum in a relatively short time, but also because of a lack of knowledge from the secondary school. To support these students, we have introduced two e-learning courses: “How to make a graph of a real function” and “How to perform the semester work”. These courses include not only the necessary theory, but also give exemplary solutions of particular problems and comments and instructions about how to solve the problem with the help of MATLAB. In order to fill in gaps from their secondary education, students can choose the subject “Mathematics for chemists” in the course of so called lifelong learning. This contains application of secondary mathematics in the chemical profession. In this way, we try to help the less well-prepared students to master the lessons of mathematics in our faculty. By all these means we try to make teaching more effective so that the reduction of lessons does not reduce the quality of the students’ learning.

Self- and Peer-Assessment in Group Work

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Abstract

This paper considers the introduction of self- and peer-assessment for group projects in two undergraduate modules, involving over 160 students, at Loughborough University. A web-based assessment system, web-pa, is used. The paper describes the reasons behind the introduction of self- and peer-assessment and the system used. It then describes the implementation of these forms of assessment and discusses the more successful and the less successful features. It is found that the students are more satisfied with the assessment process, as individual marks can be allocated for each student in a group. Moreover, the assessment of key skills, such as team working and communication, can take place. It was also found that there was no significant increase in workload for the lecturer involved and complaints by students about those who do not contribute fully, although not eliminated, were reduced. Finally, advice is given to other academics who may wish to introduce these forms of assessment.

1. Introduction

Group work is vital for engineers and therefore needs to be included within an undergraduate engineering degree. Much has been written on the use of group work (e.g. Jacques (1995) and Thorley & Gregory (1994)) and there are helpful guidelines for the introduction of group work in undergraduate mathematics in MacBean et al. (2001). However, assessment of group work raises many issues. Within group projects not only do academics wish to assess the final output, but often they also wish to assess the contributions of individual members and how the group has worked together as a team to produce the work. In many cases they end up assessing only a single final output and give all students in a group the same mark. This is often the case with large class sizes as a single final output from each group, as opposed to individual outputs, reduces the burden of marking for academics. Moreover it is difficult, if not impossible, for academics to assess individual contributions to the single final output and hence they resort to allocating each student the same mark. This can alienate students who feel that academics have not differentiated between those who have contributed much and those who have contributed little. Another unwelcome by-product of the single group mark for each individual is that the spread of marks within the class tends to be low.

Studies of current practice in a number of British Universities' Engineering Departments reveals that much of the recent work to address the issues outlined in the previous paragraph has focused on various forms of self- and peer-assessment (Barton et al. (2002)). Benefits of such forms of assessment are outlined in, for example, Choo (2003). These benefits include improved student learning through the promotion of 'deep' learning based on understanding and reflection and improved student understanding of assessment requirements. Moreover

team working, communication, interpersonal and organisational skills can be assessed. Walker (2001) found that students perceived that assessment by one's peers was fairer (than by academics) since one's peers have a greater knowledge of individual group members' contributions. However some lecturers are reluctant to implement self- and peer-assessment. There are many reasons for this, including fear of the control of assessment being taken out of their hands and the fear of over-burdening themselves with extra processing of marks.
This paper describes the introduction of self- and peer-assessment in group work for over 160 Science and Engineering Foundation (SEFS) students and first year Sports Technology (ST) students at Loughborough University during the academic year 2005/6. It explains the reasons behind the introduction of these forms of assessment and describes the implementation of them. Inter-peer assessment, where each group assesses the output of the other groups, is used together with intra-peer assessment, where students assess the contributions of their fellow group members to the project. In addition, students are asked to self-assess their own contribution, both to team working and to the final output of the project. A web-based self- and peer-assessment system, which has been developed by the Engineering Education Centre at Loughborough University, was used by students to record their scores. The scores are used to weight the project mark and hence each student receives an individual mark for the project, instead of all students in a group receiving the same mark. This paper describes and explains how the web-based system has helped in gathering and analysing the information.

Finally, this paper reports on the outcomes of the initiative. Recommendations for others wishing to adopt some or all of the features introduced here are provided.

2. Background and reasons for the introduction of self- and intra-peer-assessment

The author uses group work as part of the continuous assessment process in two undergraduate modules. The first is a module on learning and communication skills for approximately 130 SEFS students. These students work in groups of four students and research a topic from a given list. The output from the project is a poster. The second module is a mathematics module for approximately 35 first year ST students. The ST students work in groups of three and carry out Matlab projects on sports-based topics. The output from the project is a report and a poster.

Inter-peer assessment had been used successfully in previous years within the SEFS course. This involved the posters being displayed and, within their groups, students being asked to mark the posters from the other groups. In fact 20% of the marks for the posters were allocated in this way, the remaining 80% of the marks being allocated by the lecturer. This inter-peer assessment gives the students the opportunity to see the standard of work produced by other groups and also, by discussing with fellow students, they can learn to critically appraise a piece of work. The method is open to abuse as all students could decide to allocate very high marks to other groups and hence all the marks would be inflated. However the students do not adopt this approach. They approach the task with maturity and enjoy the experience of assessing the output of other students. It also offers them the opportunity to reflect upon their own work in comparison to that of their peers.

Until this academic year, the project outputs were marked by the lecturer, and also the students in the case of the SEFS course, and each student in the group received the same mark. However there was a proviso that if a student had not contributed much/anything to the group project then the lecturer could take this into account and adjust marks accordingly. However this was intended only to be used in very rare cases. It soon became apparent that a number of the students were very unhappy with the fact that all students in a group receive the same mark. This was particularly so in the case of the SEFS students who, as the projects started early in the first semester, often ended up in a group with students they did not know. In some cases there were minor complaints. However in other cases, where the group members had not worked well together, feelings ran very high about the allocation of marks. Much time was spent dealing with students who wanted to put forward the case that one or more members of their group had not contributed enough and should be marked down. Evidence from all parties had to collected and decisions reached about fairly allocating marks. It became clear that another method was needed to ensure fairness and also to lessen the amount of time spent by the lecturer in resolving disputes about contributions to the group process.

Moreover it was also apparent that the assessment process was not assessing all the learning outcomes of the modules. In particular the ability to work in a team and communicate effectively were not being assessed explicitly. In addition the marks in previous years demonstrated a very small spread, which was concerning as the varied ability of the students was not being reflected in the marks for the group projects.

Thus it was decided to introduce a system of peer-assessment. As inter-peer assessment was working well, it was decided to extend this system to intra-peer assessment, where students would mark the contribution of fellow students, within their group, to the group process and project. Self assessment would also be introduced.

Simultaneously, senior staff at Loughborough University conducted a review into group projects and the Programme Development and Quality Committee concluded in February of 2006 that, “There was a need to address student concerns about whole group marks”. Moreover it was decided that “Departmental guidelines should require group work assignments to have an element of either individual or peer assessment or both. Web-pa, although originating from Engineering, was well-documented and transparent and its use in any discipline could be encouraged to help manage self and peer assessment.”

3. Web-pa - The web-based self- and peer-assessment method at Loughborough University

Some engineering staff at Loughborough University had gained much expertise in the area of self- and peer-assessment. The staff trialled various algorithms and details can be found in Willmot and Crawford (2004). The system which was found to work best was developed into an online web-based assessment method, called web-pa. Although this system is password protected, readers can view a similar system, Sparks, which has since been developed in Australia, (http://www.educ.dab.uts.edu.au/darrall/sparksite/).

Web-pa uses a criteria based approach. Students mark themselves and others in their group against given criteria. The lecturer has the freedom to write as many or a few criteria as he/she wishes and to determine what these should be. Four criteria were set for the SEFS students. Figure 1 provides a screen shot of the criteria.
The lecturer decides when the marking criteria will be made available to students. For the modules here, the students were given a week, immediately after the submission of the project work, in which to log onto web-pa and carry out the assessment. An important point to note is that students can log onto the system on their own and, without pressure from peers, can allocate marks. Also the process is anonymous as students do not learn what marks have been allocated by fellow students.

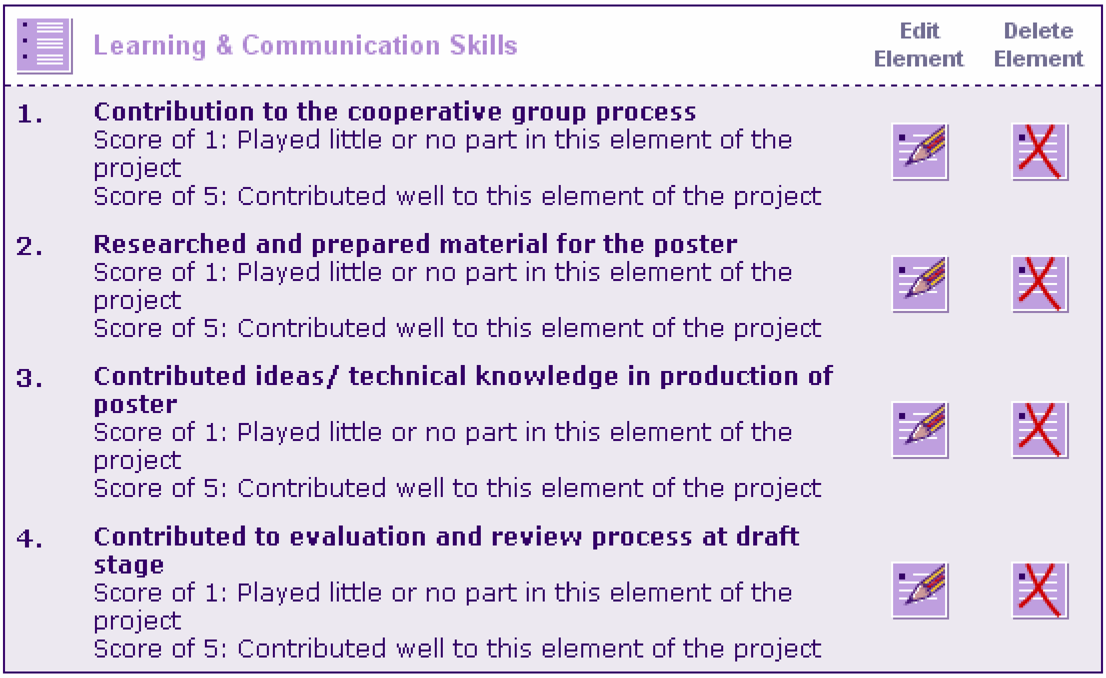

Figure 1 – Screen shot of typical criteria for self- and peer-assessment.

In the situation described in Figure 1, there is a maximum of 20 marks for each student from each fellow student and from themselves. (In a group of 4 students the maximum score is 80 for each student.) When each student's score has been calculated, a weighting factor, the web-pa score, will be allocated to that student. The web-pa score is found by dividing the student's mark by the average mark obtained by students in his/her group. The web-pa scores are then used to provide each student with an individual mark for the project. The lecturer can chose how much weight to give to the self- and peer-assessment process by deciding how much of the group project mark should be weighted by the web-pa score.

Example: Students A, B, C and D obtain marks of 32, 52, 66 and 78 in the self- and intra-peer assessment (Table 1). The average score is thus 228/4 = 57.

Student	**Marks awarded to self**	**Total marks awarded**	**WebPA score**

A	12	32	0.561
B	14	52	0.912
C	14	66	1.158
D	18	78	1.368
Grand Total of Marks		**228**	
Average Total Mark		**57**	

Table 1 – Example of output from Web-pa

The web-pa scores for each student are calculated by dividing the total mark for a student by the average total mark. Thus, for student A the web-pa score is 32/57 = 0.561. Now assume the project mark in this example is 60%. Table 2 shows the marks obtained by the four students for the situations where 30%, 50% or 100%, respectively, of the project mark is weighted by the individual web-pa score.

Percentage of project mark weighted by individual web-pa score	**Marks obtained by individual students for project mark of 60%**			
	Student A	**Student B**	**Student C**	**Student D**
30	52	58	63	67
50	47	57	65	71
100	34	55	69	82

Table 2 – Examples of individual marks obtained by each student in a group when the web-pa scores were weighted by 30,50 and 100%.

As can be seen in Table 2, the greater the percentage of the group mark weighted by the individual web-pa score, the greater the spread of marks. However it should be noted that the average of the marks for the group remains at 60%, i.e. the mark allocated for the project. This is one of the advantages of this system, in that the marks are not inflated, which is seen as a danger in other methods, see Willmott and Crawford (2004).

4. Implementation of web-pa in two undergraduate modules

The web-pa system described in the previous section was implemented by the author in two undergraduate modules in 2005-6. At the start, the author familiarised herself with the web-pa system and read though training material produced by Loughborough University's Professional Development team on self- and peer-assessment. Criteria had to be drawn up for the self- and peer-assessment and entered, along with other information about groups, dates, etc., into the system. Students were then instructed in the new system and given advice about how to complete the web-pa forms and also informed how web-pa calculated a score which would then be used to adjust the project mark for each individual. Due to lack of time within lectures, this instruction was via an email with a link to information on the module website.

Moreover the author did not have time to instruct students in self- and peer-assessment. The students were left to their own devices for this.
As students needed to be told of assessment criteria in advance of the assessment the percentage of the project mark which would be weighted by the web-pa score had to be determined in advance. Here the author took 30% as the percentage as it was decided to opt for a relatively low weighting for the first implementation, in case the assessment process did not work well.

5. Outcomes of the introduction of self- and peer-assessment

Table 3 highlights some of the findings from the introduction of the web-pa system. These are commented upon in the sections on successful and not so successful features. It should be noted that although 153 students took part in the group projects, 11 other students did not (for reasons of illness, etc.). These students have been omitted from the figures in table 3, although they were originally allocated to groups.

Module	**SEFS**	**Sports Technology**
Students	121	32
Groups	33	11
Web-pa participants	98 (81%)	25 (78%)
Students with full marks in self-assessment	62 (63% of participants)	19 (76% of participants)
Number of groups where all students obtained web-pa score of 1	4	4

Table 3 – Results of the implementation of web-pa

More Successful Features:

- *The web-pa system* - The system was robust and worked well. The author found the system straightforward to set up. The lack of queries from students indicated that they found the system easy to use.
- *No additional workload for lecturer* - The author had no collating of self- and peer-assessment marks. The web-pa score was allocated by web-pa for each student and the only additional work was in weighting the group mark accordingly and in the initial setting up of the system.
- *Participation* – Approximately 80% of the students took part in the web-pa process. These students had the opportunity to reflect upon their experience of group working and understand more of assessment requirements.
- *Student satisfaction* – Anecdotal evidence from the students indicated that they welcomed the openness of the assessment criteria, the opportunity to take part in the assessment process and the fact that individual marks are allocated to each student.

This confirms the findings of others who have used the system (Willmot and Crawford (2004)).

- *Anonymity* - The anonymity of the assessment process was welcomed by the students.
- *Spread of marks* – The allocation of individual marks for all students increased the spread of marks for the projects. This spread could have been increased further had the percentage (30%) of the project marks weighted by the web-pa score been increased.
- *Assessment of all learning outcomes of the modules* – Unlike in previous years, team working, communication, interpersonal and organisational skills were assessed.
- *Evidence of good team-working* - As a by-product of the web-pa system it became clear which students had worked well together in the groups. Students in 8 groups took the decision to award one another equal marks and so all students in each group ended up with the original mark allocated for the project. The percentage of students doing this was greater amongst the ST groups which may be expected as these students were allowed to choose who to work with, whereas the SEFS students were allocated to groups.

Less successful features:

- *Non-participation* – Approximately 20% of students did not take part in the web-pa process. This is a concern as they are not taking this opportunity to reflect upon the group work. The system is currently undergoing an upgrade and the new system can penalise a student for non-participation.
- *Full marks in self- assessment* - Approximately 66% of students who took part in the web-pa process awarded themselves full marks. At a first glance this seems disappointing as it would appear that the students are not taking the assessment process seriously. The 34% of students who did reflect honestly on their contribution and award marks accordingly are then penalised compared to others in their group who award themselves full marks. The new web-pa system will allow for self assessment to be excluded, if so desired, whereas the current system means that self- and peer-assessment take place simultaneously. However it should be noted that 35% of the students who allocated themselves full marks were in the groups where students took the decision to award everyone full marks to enable all students to obtain a web-pa score of 1.
- *Complaints re lack of participation in the project* – Student complaints regarding the marks given to students who did not contribute much to the project were slightly less than in previous years but did not disappear. In total 11 students out of the original 164 took no part in the projects and there were others whose contribution was minimal. However the current web-pa version did not allow students to allocated zero to fellow students. (The minimum was 1 mark for each criterion – see Figure 1.) This is being changed in the new system. Moreover as the percentage of the project mark which was weighted by the web-pa score was only 30% this did not allow for students who had contributed very little to get a low score. Thus the author had, as in previous years, to manually adjust marks.
- *Collusion in the awarding of marks* – As commented earlier, students in 8 groups awarded each other equal marks. When questioned about this, the students involved explained that they felt that they had all contributed equally to the project, that the

group had worked well as a team and, after discussions together, they decided to award equal marks for everyone in order that all would receive the project mark. Clearly it is heartening to hear of groups where the team working was working well. However it is unlikely that all students did in fact contribute equally and some students may have felt pressurised into this agreement. Thus, in retrospect, it may be best if this practice did not take place and if students could be discouraged from taking part in it. However in practice it would be difficult to enforce.

6. Summary and Conclusions

Self- and peer-assessment has been introduced by the author in two modules, using a web based assessment method, web-pa. There were four main reasons for introducing the system and we now summarise whether these have been addressed.

Student dissatisfaction at the unfairness of a system which allocates each group member the same mark was reduced by the introduction of web-pa, but not eliminated. Increasing the percentage weighting given to the web-pa score and allowing zero marks for non-participation would help. However it should be recognised that it is inevitable that some circumstances will not be dealt with by any system and that the lecturer will need to exercise judgment.

Extra work for the lecturer was avoided by the web-pa system which collated self- and peer-assessment marks. However the time-consuming complaints from students did not altogether disappear.

For the first time in the modules, assessment of module learning outcomes such as team-working and communication skills took place. 80% of students involved took part in the process, although not all reflected fully on their experiences. The lack of instruction to students on self- and peer-assessment has no doubt led to some of the lack of participation and to students not taking self-assessment seriously. It is suggested that much more needs to be done to teach students about the importance of reflecting on the learning experience and what can be gained from it, in order to ensure more students taking part fully.

The system does allow for a much increased spread of marks and the lecturer can decide how great this should be based on the percentage weighting given to the web-pa score.

Thus, in conclusion, the introduction has brought clear benefits to the students. The author will continue to use the system. The lessons learnt from this initial implementation have been very encouraging. In future the author would increase the weighting given to the web-pa score and allocate time to fully explain to students how the system works and the advantages of self- and peer-assessment.

Others wishing to implement a similar system are advised to consult the Sparks system, http://www.educ.dab.uts.edu.au/darrall/sparksite/.

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Widening the Model of Mathematical Studies in the Context of Lithuanian Educative Traditions

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1. Introduction

This paper presents some new ideas concerning the process of mathematical learning at higher technical educational institutions. In particular, it argues that the traditional mathematics teaching model, laying stress on the conveyance of a fixed subject system, should be replaced with a reflective study model. The essence of the latter – affording space for students to acquire knowledge and, above all, development of students' abilities via original work and personal experience. This change has been primarily stimulated by the varying the learning objectives (Bowden and Marton, 1998).

The traditional model is oriented to communicate mathematical science – status quo knowledge, acquired and generalized over many years. But, in many cases, this priority of the „status quo" knowledge makes the traditional model ambiguous and alien to the student. Present day engineers are in need of mathematical activity competence, acquired with the help of particular research elements (observation, analysis, error detection, generalization, discussion, etc.). Lecturing, the traditional and widely used teaching form, is far from being the best way to take address these skills.

Reflective study models have been explored thoroughly in the scientific literature (Kolb, 1984; Boud, Pascoe, 1978; Bowden, Marton, 1998; Boud, Keogh, Walker, 1999), but prerequisites for their realization in countries with different educational traditions represent a problem of great importance. In the context of mathematical education, solutions to this problem have not yet been fully developed.

This paper comprises two parts: firstly, the relevance of the reflective study model to organizing mathematical studies at a higher educational institution is motivated (Boud and Pascoe, 1978), and, secondly, the main limitations of the pedagogical system in Lithuania, which complicate adoption of the reflective study model to the process of mathematical studies, are discussed.

2. The reflective mathematics teaching model: reasoning on feasibility

It was established by Kolb and Fry (1975) that learning is based on constant interaction between experience and generalization of incoming information. Kolb states that effective learning is bound up with realization of the main four positions of the learning model – theory (*T*), experimenting (*E*), practice (*P*) and reflection (*R*) (Figure 1).

On the other hand, the realization process itself meets difficulties if it is not supported by the special human skills (critical thinking, information management, etc.), by facilitating communication, experimental investigation, know-how evaluation and reflection. The latter skills are indispensable to combine both the gained and the on hand knowledge, to renew and restructure one's own experience.

Kolb emphasizes that it does not matter which point is chosen as the starting point in the cycle (Fig. 1). According to him, the important thing is that all positions of the learning model should be covered. Regrettably, Kolb does not give any suggestions about the organization of the learning process, i.e., what positions of the cycle should be stimulated most of all.

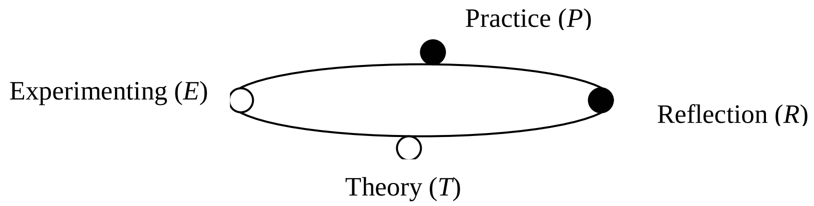

Figure 1. Kolb's learning cycle (according to Boud, Keogh and Walker, 1999)

Stimulation of the learning process and concurrent choice of the teaching model depend on the aims and objectives of the particular study modules. These aims and objectives are related to both the awareness of the educational mission and the role a particular study module plays in the curricula hierarchy. The traditional (higher) educational mission deals with accumulation of knowledge and delivery of elements of knowledge to students. Thus, when implementing the traditional mathematics teaching model, lecturers stimulate learning at positions *T* and *E*. They strive to provide students with large amounts of knowledge via lecturing (position *T*) and to consolidate the conveyed knowledge during tutorials (position *E*). Positions *P* and *R* are left to the discretion of students. This is undesirable because it is at these positions that the most complicated learning activities take place – enlargement of an individual knowledge system and passage to a new level of perceptual quality. The traditional mathematics teaching model turns out to be effective if no more than 5-10% of society are involved in higher education. Nowadays, the number of students has increased drastically (to more than 70%). So, it becomes questionable whether all students are able, on their own, to devote proper attention to positions *P* and *R*. For this reason alone, the traditional mathematics teaching model becomes ineffective.

The concept of liberal education argues that the contemporary educational mission should be oriented to the development of personal skills that allow a student to become competitive, employable and capable of managing transformations in his/her personal and social life (Barnett, 1990). Consequently, methods of mastering knowledge should be stressed rather than the knowledge itself. Knowledge will be acquired and skills developed through students' original work and practice.

Study modules on mathematics at higher educational institutions should aim to develop general intellectual skills, such as providing students with an “active” mathematical knowledge needed for their future studies. Obviously, the knowledge “activity” depends on personal knowledge structure based on the coherence of cumulative, developed and theoretical knowing. These factors highlight the importance of position *R* (Figure 1) which combines the cumulative knowledge with a particular theoretical curriculum. It is even better when theoretical generalizations, whose correctness may be checked later on, are inspired by students’ personal experience.

Reflective study models put stress on reflective learning at position *R*. Special attention is paid to the development of general intellectual skills that are achieved through analyzing, generalizing and putting personal knowledge into words (with the help of mathematical language). It is worthwhile pointing out the necessity of scientific discussions and the necessity of listening for other knowledge systems. Close reading of the relevant literature and experimenting are left to students’ own responsibility (individual work).

Any scientist, who acts in accordance with the reflective approach, goes through the same experience (Figure 2). New ideas are captured from observations of reality - observing certain laws and regularities of real world phenomena. The scientist (with the help of the objective method) gives definitions, introduces concepts and formulates hypotheses. The latter hypotheses sometimes appear to be more evident than the deductive proof itself and, consequently, they do not need any reasoning. But, mathematical science cannot be identified with an empirical one, proofs are required. A new level of abstraction is mastered through reasoning and generalizing. Thus, a new theory, not alien to a student, is developed.

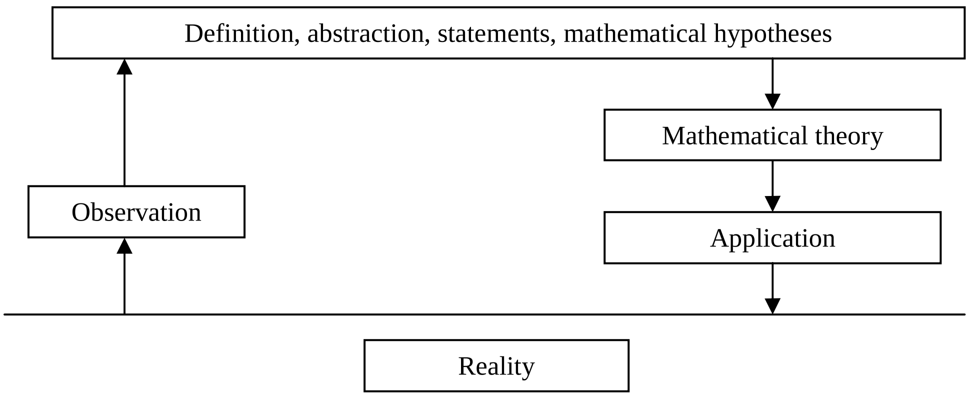

Figure 2. The ways of mathematical learning (according to J. Borwein)

Now, visualization of the mathematics learning scheme, which coincides with the reflective study model (Boud, Pascoe, 1978), becomes possible (Figure 3).

As can be seen, the whole cycle opens with an individual experimenting. The inner cycle is devoted to learning – individual activity of the student. The inner cycle parts coincide with Kolb's learning cycle positions. The main aim of the inner cycle is to form an individual knowledge structure.

The outer cycle is devoted to teaching. The very first part lays stress on student's activity that is necessary for monitoring of student's original work. It prevents the inner cycle from neglecting the learning process, because in the outer cycle students have the possibility of becoming acquainted with other learning strategies, of expressing their own knowledge, of evaluating and comparing them. The third part is necessary for conceptualizing and systematizing the material available. No lecturing is needed; the lecturer analyses students' presentations, highlights parts of the material that should undergo either revision or additional studying and assists in proving various hypotheses. As many theories are founded on facts, and new theories often originate from practical investigations (Carr and Kemmis, 1986), the last part of the model is devoted to practical applications of new theoretical findings, which, in their turn, lead to some new problems and lay foundations for a new learning cycle.

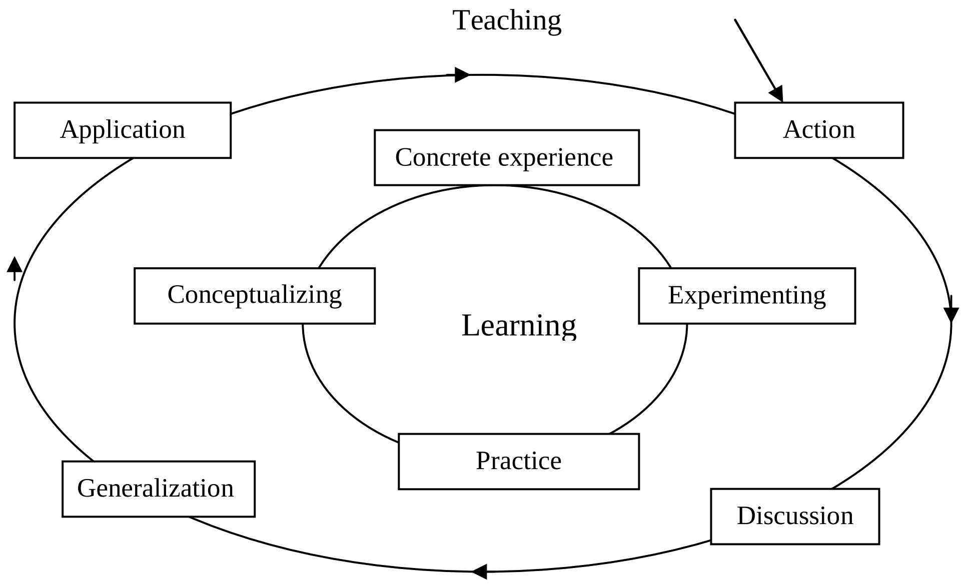

Figure 3. The empirical learning-teaching model (adapted by Boud & Pascoe, 1978)

Both cycles of the model (Figure 3) are very important in the middle of the learning process. The inner cycle develops individual knowing and skills at a hypothetical level, the outer cycle coordinates the level of individual knowledge, discovers possible mistakes (errors) and helps to raise the knowledge into a higher level, i.e., to create (individual) mathematical theory. Obviously, this theory cannot compete with the professional theoretical abstractions, nevertheless the individual theory is just what an engineer needs.

3. Constraints on the realization of the reflective study model in Lithuania

Now, considering well-established Lithuanian educational traditions, we can draw some conclusions concerning barriers to the implementation of the reflective study model in Lithuania.

A conceptual definition of the educational environment implies the following four dimensions (Ramsden, 2000):

- didactic conditions (objectives, content, methods);
- competence abilities (student's and lecturer's experience);
- psychological conditions (relationship, motivation factors);
- material facilities (textbooks, instructional aids).

The traditional teaching model is aimed at conveying more and more specialist information, whereas the reflective study model is oriented to provide students with a range of general skills. Any changes in the usable learning (educational) model require changes in the learning environment too. So, in the transition period, there might be some misunderstandings or, even, dissatisfaction among departmental staff members. Many lecturers are not inclined to accept innovations. New ideas should be always put into practice following the rules of change and laws of transformation management. Frankly speaking, a great deal of preparatory work is needed.

In preparing to change to the reflective mathematics teaching model, the very first thing to do is to narrow down the content of the respective study modules in order to leave more space for the development of students' skills and abilities. Here, a weighted compromise is desirable - the study module's content should meet (more or less) the SEFI Core Curriculum recommendations.
Secondly, giving lectures to large audiences (more than 150 students) turns out to be less motivating than small group tutorials. Again, any action in this direction (trials to reduce the size of an audience) requires considerable upheaval in the whole educational system at the university. Such a revolutionary approach is not acceptable.

The inner learning cycle (Figure 3) is realized by the student him/herself, i.e., on the basis of his/her individual studies. Obviously, the student should be provided with necessary teaching materials and training appliances. Concrete methodological guidelines, containing the detailed requirements, tasks and activities for each part of every cycle (Figure 3), should be available for the student. The learning process should be equipped with modern literature and textbooks, which highlight multi-faceted experimental possibilities and lead to knowledge-based generalizations. The whole material should be supplemented with relevant software (IT programs, graphical data, animation, etc.) to illustrate and visualize the most important topics of a particular study module. On the other hand, the training appliances should be supported by self-assessment tests, oriented to introduce, reflect and evaluate the current parts of the learning material. Finally, the inner cycle (Figure 3) could be improved with distance learning elements (WebCT environment, etc.), containing some necessary hints, keys and help.

Another problem is the student's competence. The inner cycle makes great demands of the student him/herself: individual studies, analytical reading skills and so on. Though secondary school syllabi declare that they provide schoolchildren with mathematical literacy, the reality is rather different – for instance, only 30% of students (out of 108 respondents from the Computer Science program at Kaunas University of Technology) are able to work individually, as demonstrated by their ability to answer given elementary questions. The conclusion can be made that either mathematical textbooks are difficult to understand or students are not accustomed to original work. There is a subtle problem in the school-teachers themselves (152 respondents) – only 65% of them link the skills of individual reading of mathematical textbooks with mathematical literacy. The prevalent opinion can be described this way – only the teacher (not the student) is able and bound to give explanations of mathematical themes and topics. Though textbooks on mathematics are changed and refashioned, teachers till now interpret the textbook as a source for problem solving, but not suitable for individual studies. So, the new reflective mathematics study model is limited by a student's inability to work individually, also, by the lack of learning strategies and skills.

The secondary school syllabi are heavily overloaded and schoolchildren are not mature enough to orient themselves towards the development of learning strategies.
To summarize the comments presented here, we state that the introuction of the reflective mathematics teaching model in Lithuania is limited by the following factors (limitations):

- competence limitations: the students are not accustomed to doing independent work; the students lack communication, mathematical text reading and comprehending skills; the teachers lack experience of working in a student-centered educational paradigm;
- material limitations: absence of well-prepared mathematical textbooks, task-oriented training appliances, tests varying in difficulty, etc.;
- psychological limitations: the previous experience of lecturers of mathematics, as well as their attitude towards conveying mathematical science, restricts the development and implementation of new ideas.

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Mathematical Workshops

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1. Introduction

This paper presents some information on a new form of curriculum in Mathematics organised for students on bachelor programmes at the Mechanical Engineering Faculty of Slovak University of Technology in Bratislava during the winter examination period of the academic year 2005/2006.

Many problems concerned with the mathematical education of students at technical universities appear to be more serious with the adoption of the university study programmes to conform with the new three-level structure of the Bologna declaration and the harmonisation of university studies in Europe. In the last five years, new bachelors programmes were introduced as the basic level of the university study. During the development of these programmes there were many discussions not only on what subjects should be taught at this level, but also on the optimal content and difficulty of these subjects.

Mathematics, which has always been an irreplaceable part of the basic knowledge necessary for any engineering education, suffered probably the worst of any subject, in a series of transformations of the school system. Instigating such changes seems to be a favourite activity of politicians all over the world. People actually working in these systems recognise how much harm these transformations have caused and will continue to cause and how disastrous they are in the view of the declining interest of young people in education, learning, university, science, research or any serious work and study.

Reductions in the number of lessons and practicals have led the reductions in the content of basic mathematics courses in technical university study programmes not only in Slovakia but also, it seems, throughout Europe. Following the American way of taking things if not easy then at least not so seriously, Slovakia has ended up with less than half the number of lessons, while keeping more then half content of the Mathematics course from the former five-year engineering study programmes.

As a consequence, students are the victims, suffering most and encountering serious study problems. Furthermore, secondary school graduates are not obliged to pass the final examination in Mathematics, even though they may be planning to apply to study at a technical university. Alternatively, such students may choose from two levels (A or B) of mathematics. The problems that such diversity and underpreparation cause cannot be ignored.

In the academic year 2005/2006, all universities in Slovakia observed a serious decline in the basic knowledge of university freshmen. There may be many reasons for this, however, the university academic staff who faced these limitations, had to act in order to improve the situation and keep the students' failure rate to an acceptable level.

Mathematics has become a subject in which it is fashionable to show a distinct lack of ability. It is not clear quite how this has happened since the tremendous technological development of mankind throughout history could not have taken place without the now widely unpopular science of mathematics.

This is probably the place to start. We must improve the image of Mathematics as a science among young people. We must try to change their dismissive attitude, and encourage them in their efforts to understand and consequently to appreciate mathematical knowledge that might be beautiful, interesting and useful for solving their own professional problems.

The basic idea and the dominant motive behind the introduction of the mathematical workshops was an effort to encourage students to participate in creative approaches to study during preparation for the examination in Mathematics I. We wanted to stimulate their interest in the subject and to support their individual involvement in solving problems.

2. Motivation

The main and the most common problem which university freshmen face, is a complete change from their school lifestyle. They have to become accustomed to receiving larger amounts of new information; a wider range of subject matter than they expected; a different scheme of course scheduling; and different methods and forms of teaching and assessment. This inevitably results in the necessity for them to find for themselves new methods of learning.

The classical form of support for students during the term is individual consultation with a teacher. Students in their first term rarely take up the opportunity for such consultations. The reasons for this include: low interest in the subject; low capability, low, reluctance to follow up the subject, bashfulness, unwillingness to attract the teacher's attention, and more often than not low ability in verbal expression. To improve the situation we were looking for new forms of teaching that would moderate shyness and stress and reduce the distance between a student and a teacher. We decided to offer the friendly atmosphere of a study room where expert help was available. We proposed a workshop where students would discuss problems related to Mathematics I. in mutual cooperation and if needed, with opportunity of expert consultation.

3. Implementation

The mathematical workshops were held weekly during the examination period of the winter term 2005-2006. These new consultation activities were announced by all lecturers at the last lectures of the term. Posters that invited (and encouraged) all interested parties to attend the workshops were displayed throughout the faculty building (Figure 1 shows one of these posters – translated into English). Prominent information notices were also placed on the webpage of the Department of Mathematics.

Once a week students of Mathematics I were provided with a study room and 4 or 5 teachers from the Department of Mathematics attended for 2 hours. The workshops were open to all Mathematics I students (from the daily, external and inter-branch forms of study). In the beginning the students outlined or brainstormed the themes they wanted to deal with. Some of them came prepared for the workshop and were equipped with a list of particular mathematical problems on theory or application. Others came less well-prepared, and these were helped by the teachers to understand where their problems lay. The students could choose whether to work on their own or in groups, with or without help or supervision from the teachers.

MATHEMATICAL WORKSHOP

Are you a first or the second year student and you are not sure how to pass examination form MATHEMATICS I successfully?

- do you have problems solving mathematical exercises?
- do your classmates have no time to solve them with you?
- did you not understand completely the theory and do you need some additional explanation?
- do you want to solve some mathematical problems under the guidance of someone who can help you?
- do you want to be sure that your solutions are really correct?

!! Then do not hesitate and come to the first Mathematical workshop!!

PROFESSIONAL TEAM FROM THE DEPARTMENT OF MATHEMATICS WILL HELP AND GUIDE YOU

on Wednesday, January 11, 2006 in room no. 312 from 10.00 to 12.00

You are welcome!

P.S. If you miss this opportunity.. the next chance is on Wednesday, January 18, 2006

Figure 1: Invitation poster for the 1st workshop

4. Preliminary Numbers

There were 5 workshops altogether, held during each week of the examination period, on the same day of the week. Each workshop was attended by about 11-24 students. In total, 62 students came to at least one session - this number represents 16.15% of all students examined in Mathematics I. 50 of these 62 students were from the daily study branch - 16.56% of all examined daily study students. Comparing successfulness in Mathematics I. 4 students (1 daily study) participated in the workshops but did not take the examination – these students are excluded from subsequent statistics. 84.48% of students who attended the workshops passed the exam and among the daily study students, 85.71% percents of workshop participants passed. These figures compare with pass rates amongst the students who did not attend the workshops of 62.11% overall and 60.71% of daily study students. Putting workshop participants and non-participants together, the number of students who passed the exam in Mathematics I was 67.19% overall and 66.89% for daily study students. These data are displayed in Tables 1, 2 and 3 and Figure 2.

	Daily Form of Study		All Forms of Study	
	Number	Rate	Number	Rate
Exam Attendance	302	100 %	384	100 %
Pass Exam	202	66.89 %	258	67.19 %
Workshop Attendance	50	16.56 %	62	16.15 %
Workshop Attendance prior Exam Attendance	49	16.23 %	58	15.10 %

Table 1: Attendance and Total Exam Success Rate

Workshop	Daily Form of Study		All Forms of Study	
	Number	Rate	Number	Rate
Workshop Attendance Prior Exam Attendance	49	100 %	58	100 %
Pass Exam	42	85.71 %	49	84.48 %

Table 2: Workshop Exam Success Rate

No Workshop	Daily Form of Study		All Forms of Study	
	Number	Rate	Number	Rate
No Workshop Attendance	252	100 %	322	100 %
Pass Exam	153	60.71%	200	62.11 %

Tab. 3: No Workshop Exam Success Rate

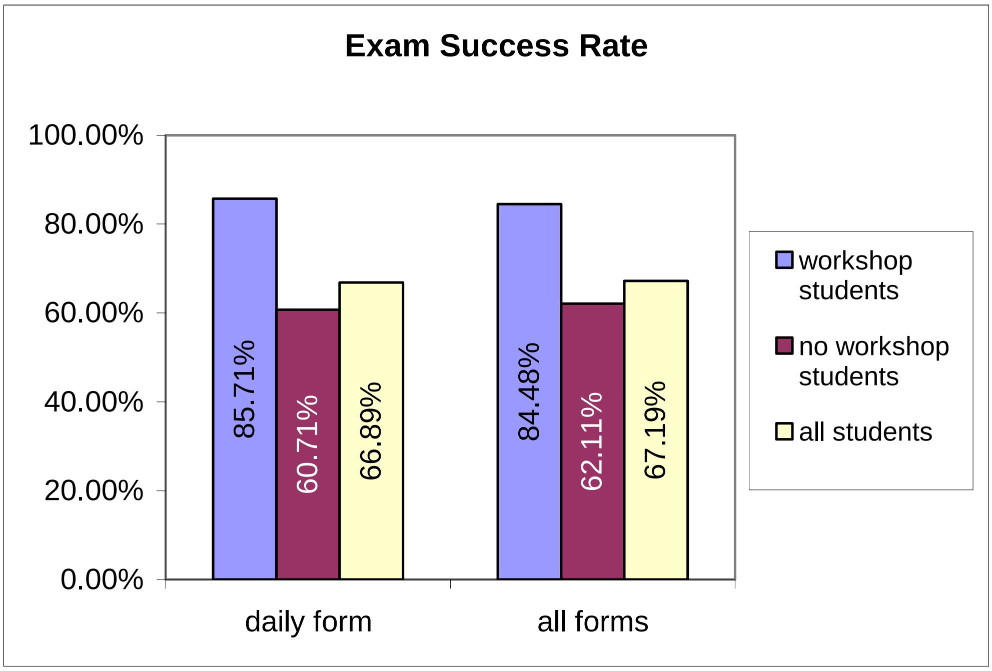

Figure 2: Exam Success Rate

5. Conclusions

Teachers participating at the workshops greatly appreciated this new form of consultation. Their evaluation was very positive with respect to both the teachers' and the students' active engagement. The atmosphere at the workshops was very friendly; students were enthusiastic about gaining new knowledge, solving problems, discussing among themselves and with their teachers. They came with a clear objective to learn, and this was the best motivation they could have. This "self-motivation" of participating students was also regarded as the most challenging professional benefit for participating teachers. The main idea of team working, brainstorming and cooperation proved to be fruitful.

Students discussed the mathematical workshops and evaluated them by themselves. Many of those, who participated, were very satisfied. The professional help provided in a discussion with teachers, was regarded as the most positive aspect of the workshops. Students definitely preferred this face-to-face approach to any available published or electronic learning materials or e-learning courses and on-line consultations. Almost all of them said they would like to have the same possibility in the summer term. Some of those who did not participate complained about the lack of information, saying that if they had known about the event they would have come. So we may expect more participants in the next semester.

In spite of the fact that mathematical workshops were heavily advertised, announced repeatedly and in due time, not all students, who might have been interested in participation, did come across this information. For these reasons we decided to call the attention of our students to this newly available form of consultations continuously throughout the whole semester. We will also provide relevant information to the student hostels and to the Students parliament, which will circulate it to those students who do not enter school regularly.

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Discussion

Discussion on integrating technology into the mathematical education of engineers

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There are several technological devices and software programs which might be integrated into the mathematical learning process. As regards hardware, there are, for example, handheld calculators with different capabilities or measurement tools providing data for further investigation. With respect to software, there is web technology (linked web material, Java applets), Computer Algebra Systems (CAS), learning and training programs, numerical software like Matlab, Dynamic Geometry Systems or application programs with some mathematical content like CAD.

It was the goal of the discussion to find out which kinds of technology are used and in what ways by the participants of the seminar. Moreover, experience concerning advantages and disadvantages should be shared and discussed. There are whole conferences devoted to the use of technology like the series of ICTMT conferences, and many questions are still the topic of research in the didactics of mathematics. So, we did not expect to get definitive answers but just wanted to get an overview of current practices and the main topics of debate.

Nearly everywhere, some material is placed on the web, including lecture notes, exercises and solutions. Some web sites provide more sophisticated tutorial support for students working on exercises. Some participants also refer their students to the mathcentre web site in the UK (http://www.mathcentre.ac.uk/; see also the contribution by S. Carpenter, T. Croft, and D. Lawson) where an enormous package of support material is offered including downloadable DVDs where mathematical topics are explained with short videos. These are in English but in Northern countries that does not seem to constitute a problem for the students.

Also, at many places, CAS packages (Maple® and Mathematica® mostly) are available via campus licenses. However, they do not seem to be used systematically in actual teaching by many lecturers. Very few departments have taken strategic decisions to force lecturers to make use of technology (or force them not to!). Consequently, the usage of technology depends heavily on the individual decisions of lecturers. If technology like CAS is just offered as an add-on that is otherwise not integrated into the teaching and learning process then it is not well accepted by students since they have a considerable work load from other subjects as well as mathematics.

At some places, Matlab® is used as a package for numerical computing, in particular when this is required by engineering colleagues. Technology seems to be more appreciated by students when it is not just used in the mathematics lectures but also in application subjects. This is often the case with Matlab® and its many toolboxes (including Simulink for design and control). As to mathematical lectures, numerical analysis is the subject where Matlab is used most often.

When mathematical programs (numerical or symbolic) are used more extensively, this often happens in project work or when working on laboratory problems. Otherwise such software is used to provide illustrations in lectures, and usually this is only feasible with small audiences. Generally, most use of technology aims at enhancing lectures rather than replacing them.

In the discussion the general attitude towards using technology was positive, but there was no overwhelming enthusiasm and several dangers were also mentioned.

In addition to providing helpful explanatory material (such as that available at mathcentre), one of the main advantages of technology is the opportunity to generate quickly many examples in order to formulate or support conjectures, and also to provide visualisations that support the formation of mental concepts. Moreover, technology provides an environment permitting experimentation, for example by changing parameters. This allows the introduction of a more experimental working style which might be nearer to the preferences of engineering students.

When introducing technology in the nineties, many lecturers hoped that passive students would become more motivated and active. However, reality showed that the situation is not as simple as that. There are some success stories where weaker students made considerable advances but with such studies it is often unclear whether the success is due to technology or to the additional attention the students received. More often it was found that technology helped to make the “good” students even better whereas the weaker ones could not perform the simplest operations by hand when they used technology for everything. Technology can be used as a “wheel chair” or as a “jet engine”, and when you use it as a wheel chair you might be unable to walk by yourself after a while whereas when you use it as a jet engine (based on a sound mathematical knowledge) it might empower you to do computations which would be unfeasible otherwise. There were different opinions on whether we should let the students decide on how they use technology or whether we should be more prescriptive. It was reported that weak students have huge problems in interpreting the output of mathematical programs properly, so that – as a consequence – it might be reasonable to allow usage of technology only once students are able to comprehend the output.

Another major point of discussion was the question of what students should really know and be able to perform for themselves in the “age of technology”. For example, in former times (30 to 40 years ago when most of the participants, including the author, were at school) long division used to be performed using log tables. Nobody does this any more because pocket calculators are available. Other more disputable examples are integration methods like integration by parts. This discussion is not specific to the mathematical education of engineers. In the scientific community of didactics of mathematics, the same question is discussed under the heading “Indispensable Manual Calculation Skills in a CAS Environment" (see: http://b.kutzler.com/article/art_indi/indisp.htm). There are certainly indisputable areas of what should be known and what does not need to be known. Multiplication by 10 should not require the use of a calculator whilst long division using logs is not needed any longer. But there is a very much disputed large "grey" area in between. Knowing certain definitions and procedures by heart has the advantage of freeing the mind for understanding new concepts when, for example, in application subjects mathematical models are developed. But given the restricted time frame, one still has to make decision on what to include in the "indispensable" part. Integration by parts was discussed as an example and there was no agreement on whether it might be necessary to do say 30 exercises by hand to really understand the method. It was also mentioned that with integration in general it is not a question of how many examples you do but rather to see that there are different methods at hand and - as opposed to differentiation – there is no straight forward analysis of expressions and subsequent application of methods but one rather has to navigate and find a strategy to use and arrange the methods in order to come to a solution. Students should write up their strategy instead of just pressing buttons on the PC.

It was also mentioned that one of the major goals of mathematical education is to get students through other courses in physics and engineering. This might serve as a criterion for deciding what to include in lectures and exercises and what should be trained to such an extent that it is known by heart since it comes up repeatedly in application lectures.