Exercise sheet 1

1. Consider the following propositional formula. For which truth valuation is it true? Write down the truth table!

a) $(x \Rightarrow y) \Rightarrow x$ b) $(x \vee y) \Leftrightarrow (y \Rightarrow x)$ c) $(x \wedge y) \Rightarrow \neg x$ d) $(x \Rightarrow (y \vee z)) \vee ((y \wedge z) \Rightarrow x)$

2. Which of the above propositional formulas are satisfiable? Which of them are tautologies? Which of them are contradictions?

3. Think about the following famous tautologial consequences. Do they make intuitive sense? Prove at least some of them.

a) $(\alpha \wedge \beta) \models \alpha$ (simplification)
b) $\alpha \models (\alpha \vee \beta)$ (addition)
c) $((\alpha \Rightarrow \beta) \wedge \alpha) \models \beta$ (modus ponens / direct reasoning)
d) $((\alpha \Rightarrow \beta) \wedge \neg\beta) \models \neg\alpha$ (modus tollens / indirect reasoning)
e) $((\alpha \vee \beta) \wedge \neg\alpha) \models \beta$ (disjunctive syllogism)
f) $(\neg(\alpha \wedge \beta) \wedge \alpha) \models \neg\beta$ (conjunctive syllogism)
g) $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \gamma)) \models (\alpha \Rightarrow \gamma)$ (hypothetical syllogism / transitivity of $\Rightarrow$)

4. Think about the following famous tautologial equivalences. Do they make intuitive sense? Prove at least some of them.

a) $(\alpha \wedge \alpha) \models\!\!\mid \alpha,\ (\alpha \vee \alpha) \models\!\!\mid \alpha$ (idempotent laws)
b) $\neg\neg\alpha \models\!\!\mid \alpha$ (double negative)
c) $(\alpha \wedge \beta) \models\!\!\mid (\beta \wedge \alpha),\ (\alpha \vee \beta) \models\!\!\mid (\beta \vee \alpha)$ (commutativity of $\wedge$ and $\vee$)
d) $((\alpha \wedge \beta) \wedge \gamma) \models\!\!\mid (\alpha \wedge (\beta \wedge \gamma)),\ ((\alpha \vee \beta) \vee \gamma) \models\!\!\mid (\alpha \vee (\beta \vee \gamma))$ (associativity of $\wedge$ and $\vee$)
e) $\neg(\alpha \wedge \beta) \models\!\!\mid \neg\alpha \vee \neg\beta,\ \neg(\alpha \vee \beta) \models\!\!\mid \neg\alpha \wedge \neg\beta$ (de Morgan's laws)
f) $(\alpha \wedge (\beta \vee \gamma) \models\!\!\mid ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma),\ (\alpha \vee (\beta \wedge \gamma) \models\!\!\mid ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)$ (distributivity laws)
g) $(\alpha \Rightarrow \beta) \models\!\!\mid (\neg\beta \Rightarrow \neg\alpha)$ (contrapositive)

5. Recall what is a set and how the basic operations with sets are defined. Try to write down a formal definition of *intersection* $\cap$, *union* $\cup$, *inclusion* $\subset$, and *equality* $=$.

6. Prove that $A \subset A \cup B$ for any pair of sets A and B.

7. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

8. In the last two exercises, you probably used some of our tautological consequences or equivalences. Try translating some of the others to the language of sets. (It might not always be possible in a reasonable way.)