Exercise sheet 3

1. Find the greatest common divisor

a) $\gcd(496, 207)$, b) $\gcd(874, 667)$.

2. Decide, whether the following equation has an integer solution

a) $x + y = 1$, b) $81x + 17y = 4$, c) $4x = 6y - 7$.

3. Find all integer solutions of the following equation

a) $10x + 14y = 32$, b) $3x + 2y = 5$, c) $10x + 15y = -3$.

Solving the following problems and presenting the solution at the blackboard, you can gain two points! However, the full two points will be awarded only if there are no serious gaps in your solution.

4. *[2 pt.]* Prove that Euclid's algorithm works without using Bézout's theorem. In particular, try to prove directly that $\gcd(a, b) = \gcd(b, r)$ for $a = kb + r$.

5. *[2 pt.]* Prove that a number $n \in \mathbb{N}$ is divisible by 3 if and only if the sum of its digits (in base 10) is divisible by 3.

6. *[2 pt.]* Recall how the proof by induction works. Prove using induction that $2+5+8+\cdots+(3n-1) = \frac{1}{2}n(3n+1)$ for every $n \in \mathbb{N}$.