Exercise sheet 6

1. Consider the relation on $\mathbb{R}$ given by xRy iff $y = |x|$. Draw in the plane $\mathbb{R}^2$ the relations R, R^{-1}, $R \circ R^{-1}$, $R^{-1} \circ R$.

2. Recall the definition of *injective*, *surjective*, and *bijective* function. Show that given a function $f\colon A \to B$, the inverse relation f^{-1} is a function if and only if f is a bijection.

3. A relation R on the closed interval $[-2, 2]$ is given by

$$xRy \qquad \text{if and only if} \qquad (x+y)^2 \leq 2(x-y)^2 - 2$$

a) Is R reflexive? Is it symmetric?
b) Decide, whether $1(R^{-1} \circ R)0$ (whether 1 is in relation with 0, where the relation considered is the composition $R^{-1} \circ R$).

4. Consider the following relation R on $\mathbb{N}$. Is it reflexive, symmetric, antisymmetric, transitive?

a) xRy if and only if $x + y \geq 50$,
b) xRy if and only if $x + y$ is even,
c) xRy if and only if $x \cdot y$ is even,
d) xRy if and only if $x + y$ is a multiple of three,
e) xRy if and only if $x \mid y$,
f) xRy if and only if $x \leq y$,
g) xRy if and only if $x \geq y$,
h) xRy if and only if $x < y$,

5. Check that the following relations are equivalences. For each of them, describe the equivalence classes.

a) $A = \mathbb{C}$, $x \sim y$ if and only if $|x| = |y|$,
b) $A = \mathbb{R}$, $x \sim y$ if and only if $x - y \in \mathbb{Z}$,
c) $A = \mathbb{Z}$, $x \sim y$ if and only if $x^2 \equiv y^2 \pmod 5$.

6. Let R, S be relations from A to B. Prove that

a) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$,
b) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$.