## Exercise sheet 8

- 1. Use Euler's theorem to calculate the remainder when dividing  $5^{676}$  by 306.
- **2.** Solve the congruence  $5^{676}x \equiv 3(2x+1) \pmod{306}$ .

**Definition.** A group  $(G, \cdot)$  is called **cyclic** if there exists an element  $a \in G$  such that  $G = \langle a \rangle$ .

- **3.** Show that  $(\mathbb{Z}_{11}^{\times}, \cdot)$  is cyclic. Find all possible generating elements  $a \in \mathbb{Z}_{11}^{\times}$ .
- **4.** Decide, whether  $(\mathbb{Z}_8^{\times}, \cdot)$  is cyclic.
- **5.** Consider the group  $(\mathbb{Z}_{17}^{\times}, \cdot)$ . Determine the order of the element  $[2] \in \mathbb{Z}_{17}^{\times}$ .

**6.** Show that  $(\mathbb{Z}_{17}^{\times}, \cdot)$  is cyclic. Find all possible generating elements  $a \in \mathbb{Z}_{17}^{\times}$ . (*Hint:* Observe that if a is not a generator, then neither its powers  $a^i$  are generators.)

**7.** For every  $d \mid 16$  find a subgroup  $H \subset \mathbb{Z}_{17}^{\times}$  of order d.