## Exercise sheet 8

1. Use Euler's theorem to calculate the remainder when dividing $5^{676}$ by 306 .
2. Solve the congruence $5^{676} x \equiv 3(2 x+1)(\bmod 306)$.

Definition. A group $(G, \cdot)$ is called cyclic if there exists an element $a \in G$ such that $G=\langle a\rangle$.
3. Show that $\left(\mathbb{Z}_{11}^{\times}, \cdot\right)$ is cyclic. Find all possible generating elements $a \in \mathbb{Z}_{11}^{\times}$.
4. Decide, whether $\left(\mathbb{Z}_{8}^{\times}, \cdot\right)$ is cyclic.
5. Consider the group $\left(\mathbb{Z}_{17}^{\times}, \cdot\right)$. Determine the order of the element $[2] \in \mathbb{Z}_{17}^{\times}$.
6. Show that $\left(\mathbb{Z}_{17}^{\times}, \cdot\right)$ is cyclic. Find all possible generating elements $a \in \mathbb{Z}_{17}^{\times}$. (Hint: Observe that if $a$ is not a generator, then neither its powers $a^{i}$ are generators.)
7. For every $d \mid 16$ find a subgroup $H \subset \mathbb{Z}_{17}^{\times}$of order $d$.

