Sample midterm test

The midterm test will take place on 23 November 2022. You will have 50 minutes for writing it. Its structure will be as follows:

1. *(5 b.)* Logic (either propositional or predicate, cf. HW01, 02)

2. *(5 b.)* Relations (cf. HW06) or Operations (cf. HW07(2))

3. *(5 b.)* ⎫ Transforming a number to a different base, determining the remainder when dividing,

solving a Diophantine equation (directly a Diophantine equation or a congruence or

4. *(5 b.)* ⎭ looking for an inverse in $(\mathbb{Z}_n, \cdot)$ or similar). See all the other homeworks.

The problems will look similar as those that were assigned for homeworks. The content is everything up to week 8.

Try solving the sample tests on the following page before Exercise session 9. Remember that there will be the time limit 50 minutes, so try to do it on time. We will go through the correct solution on the exercise session.

Sample test A

- The time limit is 50 minutes
- Do not forget to clearly write the answer to every question.
- All your computations and derivations should be clear or properly explained

1. *(5 b.)* Consider the following formula of propositional logic

$$((\neg a \vee b) \Leftrightarrow c) \Rightarrow ((b \wedge c) \wedge a)$$

Write down the truth table. Decide, whether the formula is satisfiable, whether it is a tautology or a contradiction.

2. *(5 b.)* A relation R on the closed interval $[1,5] \subset \mathbb{R}$ is given by

$$xRy \quad \text{if and only if} \quad (x-y)^2 \leq (2x-y)^2 + 1.$$

Decide, whether R is reflexive and symmetric. Decide, whether $1(R \circ R)5$.

3. *(5 b.)* Write down the number $n = 300$ in base 6.

4. *(5 b.)* Find all $x \in \mathbb{Z}$ that satisfy $5^{22}x \equiv 17 - x \pmod{11}$

Sample test B

- The time limit is 50 minutes
- Do not forget to clearly write the answer to every question.
- All your computations and derivations should be clear or properly explained

1. *(5 b.)* Consider the following formula of predicate logic. Construct its negation and reformulate it in such a way that the sign $\neg$ appears only in front of the "atomic" formulas only.

$$[(\forall y \, (R(x,y) \wedge P(y))) \Rightarrow (\forall x \, \forall y \, Q(x,y))] \wedge (\forall x \, Q(x,a))$$

2. *(5 b.)* An operation $\bullet$ on $\mathbb{R}$ is given by

$$x \bullet y = \sqrt{x^2 + y^2}.$$

Decide, whether $(\mathbb{R}, \bullet)$ forms a monoid. If yes, find all invertible elements in this monoid.

3. *(5 b.)* Determine the remainder when dividing 12^{97} by 35.

4. *(5 b.)* Find the inverse of $[7]$ in $(\mathbb{Z}_{123}, \cdot)$.