Exercise sheet 12

1. How many graphs on the vertex set $V = \{1, \dots, n\}$ are there?

2. Suppose c_n is the number of all connected graphs on the vertex set $V = \{1, \dots, n\}$. Show that

$$n \cdot 2^{\binom{n}{2}} = \sum_{k=1}^{n} \binom{n}{k} k c_k 2^{\binom{n-k}{2}}.$$

3. Write down the adjacency matrices for graphs K_n, P_n, S_n, C_n. (Try at least $n = 5$.)

4. Draw the *butterfly graph* that is given by the adjacency matrix

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

(If you are not sure, you can check your answer on wikipedia.)

5. In the butterfly graph above. Decide, whether the following exists.

a) A path that runs through all vertices.
b) A cycle that runs through all vertices.
c) A path that runs through all edges.
d) A trail that runs through all edges.
e) A circuit that runs through all edges.

6. Is the butterfly graph connected? Consider its subgraph induced by the vertex set $\{1, 2, 4, 5\}$. Is it connected?

7. Consider the graph given by $V = \{\text{all people born after 1900}\}$. Two people are connected with an edge if one is a child of the other. (So, we get an unoriented version of the example from lecture.) Is the graph connected? What is represented by the connected components? Do you think that such a graph contains any cycle?