Exercise sheet 13

1. Find the minimal spanning tree of the weighted graph, where the weights are given as in the following matrix (– denotes no edge, a number denotes the weight)

$$
\begin{pmatrix}
- & 9 & 13 & 5 & - & 7 & - & 9 \\
9 & - & 13 & - & 4 & - & - & 14 \\
13 & 13 & - & 6 & 5 & 4 & 9 & 2 \\
5 & - & 6 & - & 4 & - & - & - \\
- & 4 & 5 & 4 & - & 5 & - & 1 \\
7 & - & 4 & - & 5 & - & 15 & 8 \\
- & - & 9 & - & - & 15 & - & 7 \\
9 & 14 & 2 & - & 1 & 8 & 7 & -
\end{pmatrix}
$$

2. Decide, whether there is a weighted graph that

   a) has a unique minimal spanning tree,
   b) has precisely two minimal spanning trees.

3. Consider the directed graph given by the following adjacency matrix ($A_{ij} = 1$ means that (i, j) is an edge, so $i \to j$). Find the topological order of its vertices.

$$
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

4. Consider the directed graph $G = (V, E)$ with vertex set $V = \{1, \ldots, 12\}$ and edge set

$$E = \{(1, 3), (2, 1), (2, 6), (2, 7), (2, 9), (2, 12), (3, 4), (4, 8), (4, 11), (5, 3), (5, 8),$$
$$(6, 4), (6, 7), (6, 9), (6, 10), (6, 12), (7, 1), (7, 5), (7, 10), (8, 3), (11, 3), (11, 10), (12, 8)\}.$$

Decide whether it is acyclic.