

Exercise sheet 14

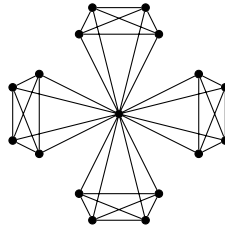
1. Give an example of a simple directed graph which has 9 vertices, 12 directed edges, 2 components of connectivity, and 4 strongly connected components.

2. Let G be a strongly connected directed graph on n vertices. Give the smallest and biggest numbers of edges that G can have.

3. Determine the strictly connected components of the following graph. How does its condensation look like?



4. Decide, whether the following graph is Eulerian. If yes, find the corresponding Eulerian cycle.



5. The *Windmill graph* $Wd(k, n)$ is a graph on $n(k - 1) + 1$ vertices constructed by taking n copies of the full graph K_{k-1} and adding an additional vertex that is connected to all other vertices. (The graph from the previous problem is $Wd(5, 4)$.) For which k and n is the Windmill graph Eulerian?

6. Consider the graph $G = (V, E)$, where $V = \{1, \dots, 8\}$ and

$$E = \{\{1, 2\}, \{1, 4\}, \{2, 3\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 8\}, \{5, 6\}, \{5, 7\}, \{6, 7\}, \{6, 8\}\}.$$

Determine, whether it has an Eulerian cycle or an Eulerian trail. If yes, find it.