Exercise sheet 2

1. Write formulas of predicate logic corresponding to the following sentences. Use the predicate symbols mentioned in the text.

a) Somebody has a musical ear (E) and somebody does not.
b) Some children (C) do not like chocolate (H).
c) Not every talented painter (P) exhibits their pictures in the National Gallery (G).
d) Only students (S) can buy cold suppers (C).
e) Not every person (P) who has expensive skis (E), is a bad skier (B).

(For instance, in (b), we denote by $C(x)$ the predicate "x is a child" and by $H(x)$ the predicate "x likes chocolate".)

2. Formulate negations of the formulas above. Both using symbols as well as in natural English.

3. For the following quantified propositions, find their negation. Simplify it in such a way that the negation symbol $\neg$ appears only in front of atomic formulas (like $P(x)$, $R(x, y)$).

a) $\forall x\, [P(x) \Rightarrow (\exists y\, (P(y) \wedge R(x, y)))]$
b) $P(a) \vee [\exists z\, (P(z) \wedge \forall y\, (R(y, z) \Rightarrow \neg P(y)))]$
c) $R(a) \wedge \forall x(Q(x, a) \Rightarrow \exists y(R(y) \wedge Q(x, y)))$
d) $\forall x\, (P(x) \Rightarrow \exists y\, (Q(x, y) \wedge P(y)))$

4. Think about the following rules for set operations. Do they make intuitive sense? Prove at least some of them. (Suppose A, B, C are subsets of some universe U and denote $A^{\complement} = U \setminus A$.)

a) $A \cup A = A,\ A \cap A = A$ (idempotent laws)
b) $(A \cup B) \cup C = A \cup (B \cup C),\ (A \cap B) \cap C = A \cap (B \cap C)$ (associative laws)
c) $A \cup B = B \cup A,\ A \cap B = B \cap A$ (commutative laws)
d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C),\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (distributive laws)
e) $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement},\ (A \cap B)^{\complement} = A^{\complement} \cup B^{\complement}$ (De Morgan's laws)
f) $A \cup (A \cap B) = A,\ A \cap (A \cup B) = A$ (absorption laws)
g) $A \cup \emptyset = A,\ A \cap U = A$ (identity laws)
h) $A \cup U = U,\ A \cap \emptyset = \emptyset$ (domination laws)
i) $A \cup A^{\complement} = U,\ A \cap A^{\complement} = \emptyset$ (complement laws)
j) $(A^{\complement})^{\complement} = A$ (double complement)
k) $\emptyset^{\complement} = U,\ U^{\complement} = \emptyset$