

## Exercise sheet 6

1. Consider the relation on  $\mathbb{R}$  given by  $xRy$  iff  $y = |x|$ . Draw in the plane  $\mathbb{R}^2$  the relations  $R$  and  $R^{-1}$ .
2. Consider the sets  $A = \{e, f, g, h\}$ ,  $B = \{1, 2, 3, 4, 5\}$ . For the following relations, draw a suitable graphical representation of these relations.
  - a)  $R = \{(e, 2), (e, 3), (e, 4), (f, 1), (g, 3), (g, 4)\} \subset A \times B$ .
  - b)  $S \subseteq B \times A$  defined by  $nSl$  if the word for number  $n$  contains letter  $l$ .
  - c)  $T \subseteq B \times B$  defined by  $nTm$  if  $2n \leq m$ .

3. Decide, whether the relation  $T$  from before is reflexive, symmetric, antisymmetric, or transitive.

For two relations  $R \subseteq A \times B$  and  $S \subseteq B \times C$ , we define their **composition**

$$R \circ S = \{(x, z) \mid (\exists y \in B)(xRy \wedge ySz)\} \subset A \times C.$$

4. Find

- a)  $R \circ S$ ,                      b)  $S \circ R$ ,                      c)  $T \circ T$ ,                      d)  $R \circ T$ ,                      e)  $T \circ S$ .

5. Consider the following relation  $R$  on  $\mathbb{N}$ . Is it reflexive, symmetric, antisymmetric, transitive?

- a)  $xRy$  if and only if  $x + y \geq 50$ ,
- b)  $xRy$  if and only if  $x + y$  is even,
- c)  $xRy$  if and only if  $x \cdot y$  is even,
- d)  $xRy$  if and only if  $x + y$  is a multiple of three,
- e)  $xRy$  if and only if  $x \leq y$ ,
- f)  $xRy$  if and only if  $x \mid y$ ,
- g)  $xRy$  if and only if  $x \geq y$ ,
- h)  $xRy$  if and only if  $x < y$ ,

6. Check that the following relations are equivalences. For each of them, describe the equivalence classes.

- a)  $A = \mathbb{C}$ ,  $x \sim y$  if and only if  $|x| = |y|$ ,
- b)  $A = \mathbb{R}$ ,  $x \sim y$  if and only if  $x - y \in \mathbb{Z}$ ,