

Exercise sheet 7

1. Decide, whether 17 is invertible in (\mathbb{Z}_{45}, \cdot) . If yes, find its inverse.

2. Find all invertible elements in

a) (\mathbb{Z}_{13}, \cdot)

b) (\mathbb{Z}_{15}, \cdot)

How many are there?

3. Consider the binary operation of *taking the average* in \mathbb{R} . That is, for $x, y \in \mathbb{R}$, define the operation

$$x \bullet y = \frac{x + y}{2}.$$

Decide, whether (\mathbb{R}, \bullet) forms a semigroup.

4. Take any set A and consider the operation

$$x \star y = x.$$

Decide, whether (A, \star) forms a semigroup. Does it have a neutral element?

5. Consider a set U and its *power set* $\mathcal{P}(U) = \{A \mid A \subseteq U\}$. Decide, whether $(\mathcal{P}(U), \cap)$ and $(\mathcal{P}(U), \cup)$ form semigroups and whether they have neutral elements.

6. Consider the set $A = \mathbb{R} \setminus \{0\}$ and the operation

$$x \odot y = \frac{1}{3}xy.$$

Decide, whether (A, \odot) forms a group.