Exercise sheet 8

1. Use Euler's theorem to calculate the remainder when dividing 5^{676} by 306.

2. Solve the congruence $5^{676}x \equiv 3(2x+1) \pmod{306}$.

Definition. A group $(G, \cdot)$ is called **cyclic** if there exists an element $a \in G$ such that $G = \langle a \rangle$.

3. Show that $(\mathbb{Z}_{11}^{\times}, \cdot)$ is cyclic. Find all possible generating elements $a \in \mathbb{Z}_{11}^{\times}$.

4. Decide, whether $(\mathbb{Z}_{8}^{\times}, \cdot)$ is cyclic.

5. Consider the group $(\mathbb{Z}_{17}^{\times}, \cdot)$. Determine the order of the element $[2] \in \mathbb{Z}_{17}^{\times}$.

6. Show that $(\mathbb{Z}_{17}^{\times}, \cdot)$ is cyclic. Find all possible generating elements $a \in \mathbb{Z}_{17}^{\times}$. (*Hint:* Observe that if a is not a generator, then neither its powers a^i are generators.)

7. For every $d \mid 16$ find a subgroup $H \subset \mathbb{Z}_{17}^{\times}$ of order d.