Exercise sheet 13

1. Find the minimal spanning tree of the weighted graph, where the weights are given as in the following matrix (– denotes no edge, a number denotes the weight)

$$\begin{pmatrix} - & 9 & 13 & 5 & - & 7 & - & 9 \\ 9 & - & 13 & - & 4 & - & - & 14 \\ 13 & 13 & - & 6 & 5 & 4 & 9 & 2 \\ 5 & - & 6 & - & 4 & - & - & - \\ - & 4 & 5 & 4 & - & 5 & - & 1 \\ 7 & - & 4 & - & 5 & - & 15 & 8 \\ - & - & 9 & - & - & 15 & - & 7 \\ 9 & 14 & 2 & - & 1 & 8 & 7 & - \end{pmatrix}$$

2. Consider the directed graph given by the following adjacency matrix ($A_{ij} = 1$ means that (i, j) is an edge, so $i \to j$). Find the topological order of its vertices.

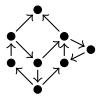
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

3. Consider the directed graph $G = (V, E)$ with vertex set $V = \{1, \ldots, 12\}$ and edge set

$$E = \{(1,3), (2,1), (2,6), (2,7), (2,9), (2,12), (3,4), (4,8), (4,11), (5,3), (5,8),$$
$$(6,4), (6,7), (6,9), (6,10), (6,12), (7,1), (7,5), (7,10), (8,3), (11,3), (11,10), (12,8)\}.$$

Decide whether it is acyclic.

4. Determine the strictly connected components of the following graph. How does its condensation look like?



5. Give an example of a simple directed graph which has 9 vertices, 12 directed edges, 2 components of connectivity, and 4 strongly connected components.