

Homework 1B – solution

1. Consider the following formula of propositional logic. Decide whether the formula is satisfiable; if yes, give all truth valuations in which it is true.

- a) $((a \Leftrightarrow b) \Rightarrow a) \Rightarrow ((b \wedge c) \vee \neg a)$,
 b) $((\neg a \vee b) \Leftrightarrow c) \Rightarrow ((b \wedge c) \wedge a)$.

We just write down the truth tables. For (a), denote $\alpha = \underbrace{((a \Leftrightarrow b) \Rightarrow a)}_{\varphi} \Rightarrow \underbrace{((b \wedge c) \vee \neg a)}_{\psi}$. Then:

a	b	c	φ	ψ	α
F	F	F	F	T	T
F	F	T	F	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	T	F	F
T	F	T	T	F	F
T	T	F	T	F	F
T	T	T	T	T	T

So, the formula is satisfiable and the corresponding truth valuations are given by the table above. (We can also say that α is true if and only if a is false or all a, b, c are true.)

For (b), denote $\beta = \underbrace{((\neg a \vee b) \Leftrightarrow c)}_{\mu} \Rightarrow \underbrace{((b \wedge c) \wedge a)}_{\nu}$. Then:

a	b	c	μ	ν	β
F	F	F	F	F	T
F	F	T	T	F	F
F	T	F	F	F	T
F	T	T	T	F	F
T	F	F	T	F	F
T	F	T	F	F	T
T	T	F	F	F	T
T	T	T	T	T	T

So, the formula is satisfiable and the corresponding truth valuations are given by the table above.