Homework 5B – solution

1. Express 160 in base

a) 3 b) 5

We follow our algorithm. Let's start with (a). We begin with writing down all the powers of 3 until we found one, which is higher than the number $n = 160$.

$$3^0 = 1, \quad 3^1 = 3, \quad 3^2 = 9, \quad 3^3 = 27, \quad 3^4 = 81, \quad 3^5 = 273$$

Clearly, $3^4 \leq 160 < 3^5$. Now, we do the division:

$$\begin{aligned} 160 &= 1 \cdot 81 + 79 \\ 79 &= 2 \cdot 27 + 25 \\ 25 &= 2 \cdot 9 + 7 \\ 7 &= 2 \cdot 3 + 1 \end{aligned}$$

Hence $160 = 1 \cdot 3^4 + 2 \cdot 3^3 + 2 \cdot 3^2 + 1 \cdot 3^1 + 1 = (12221)_3$.

Similarly, we obtain $160 = 1 \cdot 125 + 1 \cdot 25 + 2 \cdot 5 + 0$, so $160 = (1120)_5$.

2. Compute the remainder when dividing

a) 13^{35} by 7 b) 18^{45} by 23

For (a), notice that $13 \equiv -1 \pmod 7$, so $13^i \equiv (-1)^i \pmod 7$. In particular $13^{35} \equiv -1 \equiv 6 \pmod 7$, so the remainder is 6.

For (b), we can use little Fermat's theorem that says $a^{p-1} \equiv 1 \pmod p$ if p is a prime and $a \perp p$. In this case $p = 23$ is a prime, so $18^{22} \equiv 1 (\mathrm{mod}\, 23)$. Consequently, $18^{44} \equiv 1 (\mathrm{mod}\, 23)$ and $18^{45} \equiv 18 \pmod{23}$.