Homework 7A

This is a voluntary homework. Solving it, you can gain extra points to the exam. Hand in before the next lecture i.e. 8 Nov 16:15 (either on my desk in the classroom or send to my e-mail). You are eligible for getting points only if you hand in on time and only if it is solved (more or less) correctly.

Problem. Suppose $(G, \cdot)$ is a group and $\equiv$ is an equivalence on G such that

$$a \equiv b \wedge c \equiv d \quad \Rightarrow \quad a \cdot c \equiv b \cdot d.$$

Prove that $G/{\equiv}$ is a group with respect to the operation $[x] \cdot [y] = [x \cdot y]$. (Do not forget to prove that such an operation is well defined.) Suppose e is the unit in G. Show that $[e]$ is a *subgroup* of G, that is, if $x, y \in [e]$, then $x \cdot y \in [e]$.