Homework 8B – solution

1. Find the remainder when dividing 2022^{1000} by 55.

We use the Euler's formula. First, we check that 55 is coprime with 2022. Since $55 = 5 \cdot 11$, it is enough to check that 2022 is not divisible by 5 and 11. It is clearly not divisible by 5 since it does not end by 0 or 5. It is also not divisible by 11 since the alternating sum of digits is not. (Alternatively, you might also use Euclid's algorithm to find that.)

Now since $55 = 5 \cdot 11$, we have $\phi(55) = 4 \cdot 10 = 40$. So, by Euler's theorem $2022^{40} \equiv 1 \pmod{55}$. Raising this to the power 25, we get $2022^{1000} \equiv 1 \pmod{55}$.

2. Find all invertible elements in $(\mathbb{Z}_{12}, \cdot)$.

We know that the invertible elements of $(\mathbb{Z}_n, \cdot)$ are exactly those classes $[i]$ with $1 \leq i < n$ such that $i \perp n$. In this case, those are 1, 5, 7, 11. A good way how to check that you did not forget anything is to compute $\phi(n)$, which should give the number of integers coprime with a given one. In this case $12 = 2^2 \cdot 3$, so $\phi(12) = (2^2 - 2^1)(3 - 1) = 4$, which seems correct.