Homework 11B − solution

1. Consider the set $A = \{1, 2, 3, \ldots, 10\}$. How many subsets of A have exactly four elements?

We are exactly counting k-combinations ($k = 4$) of the set A having 10 elements. We have a formula for that: the number is given by $\binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$.

2. Suppose a password must contain 6 characters chosen from 26 letters of Latin alphabet and 10 digits. In addition, the password must contain at least one letter and at least one digit. How many possibilities are there?

There are exactly 36^6 strings of length 6 chosen from the total 36 characters (counting k-permutations with repetitions). We have to rule out the possibility of having 6 digits or 6 letters. For the first, we have 10^6 possibilities, for the second 26^6 possibilities. So, the final answer is $36^6 - 10^6 - 26^6 = 1\,866\,866\,560$.