Homework 12B – solution

1. Describe how positional number systems work. Formulate an algorithm for writing a given number n in base $q \geq 2$.

First, I asked, how PNS work. You sometimes tried to somehow capture the idea with a couple of sentences, which is good, but the most important thing is to really precisely define how it works, what are the digits and so on. So, you can write something like

> Let us fix a **base** $q \in \mathbb{N}$, $q \geq 2$. Then for every number $n \in \mathbb{N}_0$, we can find numbers $k \in \mathbb{N}$ and $a_0, \ldots, a_k \in \{0, 1, \ldots, q-1\}$ called **digits** such that $n = a_0 + a_1 q + \cdots + a_k q^k$. We denote it by $n = (a_k a_{k-1} \cdots a_1 a_0)_q$.

Secondly, I asked for the algorithm. This is formulated precisely in the notes I've put on my website, so you can check it there. You can also formulate it in a different way, in your own words. See below. But in any case, it should be written in such a way that anybody can go through the steps of the algorithm and without thinking too much obtain the correct answer even if they have no idea about PNS. (This is also true for the definitions by the way.) The algorithms in your homeworks were often quite incomprehensible.

1. Compute all the powers q^i of q for $i = 0, 1, 2, \ldots$ until you find k such that $q^k \leq n < q^{k+1}$.
2. Put $r_{k+1} := n$
3. For every $j = k, k-1, \ldots, 0$ do: Perform the integer division of r_{j+1} by q^j: Find a_j and r_j such that $r_{j+1} = a_j q^j + r_j$
4. Return $n = (a_k a_{k-1} \cdots a_1 a_0)_q$.

As I said, see also notes on my website. If you are struggling with writing an abstract definition/theorem, at least try explaining it on some example. (But you won't get full points for that.)

2. Give the formula for counting k-combinations of n-element set. If you use some special symbols apart from $+$, $-$, $\cdot$, $/$, define them!

The formula is

$$P(n,k) = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1}.$$

You might also write $P(n,k) = \binom{n}{k}$, but then you have to explain how $\binom{n}{k}$ is defined – either as above or using factorials $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. But if you use the factorial, you should also explain what that means: $n! = n(n-1)\cdots 1$.

3. Define, what does it mean that a graph is *connected*. Decide, whether the graph given by the following adjacency matrix is connected. Determine the number of its components.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

A graph $G = (V, E)$ is said to be **connected** if, for every $v, w \in V$, there is a path from v to w.
The graph above is disconnected having three components $\{1, 2, 3\}$, $\{4, 5\}$, $\{6, 7\}$.