DEN Homework # 1

Solve the problems and then bring your work to the lab in the second week of school.

1. For the equation $y' = \dfrac{2xy}{x^2-1}$ find a general solution
and then the particular solutions for the following initial conditions:

a) $y(0) = 1$; b) $y(2) = -3$; c) $y(-3) = 16$; d) $y(2) = 0$.

2. For the equation $y' = \dfrac{e^x}{e^y}$ find a general solution
and determine its typical asymptotic rate of growth at infinity.

Solution

1. Important observation: Restriction $x \neq \pm 1$.
1) General solution: Attempt at separation leads to stat. solution $y(x) = 0$.
Separation and integration yields $\ln|y| = \ln|x^2-1| + C$, then $y = \pm e^C(x^2-1)$, apply our favourite trick $\pm e^C = D \neq 0$.
Validity? We have to check on the equation ($x \neq \pm 1$), formula for $y(x)$ (no problem) and derivative (no problem).
Setting $D = 0$ we also include the stationary solution, so a general solution is $y(x) = D(x^2-1)$, $x \neq \pm 1$.
Three possible intervals for solution, we choose based on initial condition so that x_0 is in the chosen interval.
2) Initial conditions:
a) $y_a(x) = 1 - x^2$, $x \in (-1, 1)$.
b) $y_b(x) = 1 - x^2$, $x \in (1, \infty)$.
This is actually a distinct solution from a). Formula is the same, but this one is happenind at different time, it is an independent process.
c) $y_c(x) = 2(x^2-1)$, $x \in (-\infty, -1)$.
b) $y_d(x) = 0$, $x \in (1, \infty)$. We get this either by finding $D = 0$, or recalling the stationary solution.

2. No restriction from the equation. Routine separation:
$$\frac{dy}{dx} = \frac{e^x}{e^y} \implies e^y dy = e^x dx \implies \int e^y dy = \int e^x dx \implies e^y = e^x + C,$$
hence the general solution is $y(x) = \ln(e^x + C)$, $e^x + C > 0$. This cannot be simplified further.

When x grows really huge, then the C becomes sooner or later negligible compared to e^x and thus it can be ignored. Therefore the answer is:
For $x \sim \infty$ we have $y(x) \sim \ln(e^x) = x$.