DEN Homework 01 – solution

1. Important observation: Restriction $x \neq \pm 1$.
1) General solution: Attempt at separation leads to stationary solution $y(x) = 0$, $x \neq \pm 1$.
Separation and integration yields $\ln|y| = \ln|x^2 - 1| + C$, then $y = \pm e^C(x^2 - 1)$, apply our favourite trick $\pm e^C = D \neq 0$.
Setting $D = 0$ we also include the stationary solution, so a general solution is $y(x) = D(x^2 - 1)$, $x \neq \pm 1$.
Three possible intervals for solutions, for a given initial condition we choose the interval containing x_0.
2) Initial conditions:
a) $y_a(x) = 1 - x^2$, $x \in (-1, 1)$.
b) $y_b(x) = 1 - x^2$, $x \in (1, \infty)$.
This is not the same as a). While the formulas agree, these two take place at different times.
c) $y_c(x) = 2(x^2 - 1)$, $x \in (-\infty, -1)$.
b) $y_d(x) = 0$, $x \in (1, \infty)$. We get this either by finding $D = 0$, or recalling the stationary solution.

2. No restriction from the equation. Routine separation:
$$\frac{dy}{dx} = \frac{e^x}{e^y} \implies e^y dy = e^x dx \implies \int e^y dy = \int e^x dx \implies e^y = e^x + C,$$
hence the general solution is $y(x) = \ln(e^x + C)$, $e^x + C > 0$. This cannot be simplified further.

When x grows really huge, then the C becomes sooner or later negligible compared to e^x and thus it can be ignored. Therefore the answer is:
For $x \sim \infty$ we have $y(x) \sim \ln(e^x) = x$.