DEN Homework 02 – solution

1. From the equation $y \neq 0$. Separation: $\displaystyle\int 2y\,dy = \int dx \implies y^2 = x + C$.

Trick: $y(x) = \pm\sqrt{x+C}$. Condition $x + C \geq 0$ from the solution and $x + C \neq 0$ from the equation, hence a general solution $y(x) = \pm\sqrt{x+C}$, $x + C > 0$.

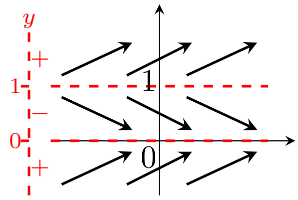
So there are two groups of solutions, we will have to choose. The sign of y_0 decides.

a) $y_a(x) = \sqrt{x+3}$, $x \in (-3, \infty)$; (the choice $y(x) = -\sqrt{x+3}$ obviously cannot yield 2)

b) $y_b(x) = -\sqrt{x-1}$, $x \in (1, \infty)$; (the choice $y(x) = \sqrt{x-1}$ obviously cannot yield -1)

c) $y_c(x) = \sqrt{x+10}$, $x \in (-10, \infty)$.

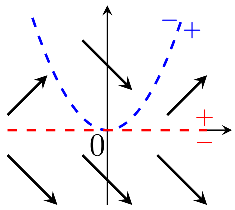
2. No restriction. It is an autonomous equation, so the sign of the right-hand side depends only on y, a one-dimensional task. We rewrite the equation as $y' = y^3(y-1)$.



We see two dividing points for the one-dimensional problem, $y = 0, 1$, in the plane these define two horizontal lines. We readily deduce signs for the strips, say by substituting suitable values for y, and we are done.

The equation $y' = 0$ has solutions $y = 0, 1$, so these are the equilibria. From the slope field: $y_0 = 0$ is stable, $y_0 = 1$ unstable. It is good to know to answer this way as well: The stationary solution $y(x) = 0$, $x \in \mathbb{R}$ is stable, the stationary solution $y(x) = 1$, $x \in \mathbb{R}$ is unstable.

3.



No restriction from the equation. Product of factors simplifies our work.

General approach: $y' = 0$ yields $y = 0$ or $x^2 - y = 0$, two dividing curves (the first one leads to a stationary solution BTW). The option „y' does not exist“ does not yield anything. We draw both curves, the second one is $x^2 - y = 0$, that is, the standard parabola $y = x^2$. We find signs for the resulting regions, say by substituting suitable points, and then draw arrows.

Is there an easier way? Due to the second factor it is impossible to separate the influences of x and y, so it will not be that simple. However, it is possible to determine easily the influence of each factor individually, see the sign marks in the picture. These are determined by reasoning or by substituting suitable points again. For instance, above the parabola we get a negative sign (try, say, the point $(0, 1)$), and positive below the parabola (say $(1, 0)$). The influence of the factor y is obvious. Then we merge the two influences.

Stationary solutions: Can we make the derivative equal to zero everywhere just by setting y so a specific value? Yes, by choosing $y = 0$. Thus $y(x) = 0$, $x \in \mathbb{R}$ is a stationary solution.

The choice $y = x^2$ would also make y' zero, but it is not a constant (stationary) function, so not applicable.

Remark: This equation is neither separable nor linear.