DEN Homework 03 – solution

1. 1. $f'(1) \approx \dfrac{(a+h)^4 - a^4}{h} = \dfrac{1.01^4 - 1^4}{0.01} = 4.060401.$

We know that $[x^4]'(1) = 4$. Thus $E_x = x - \hat{x} = -0.060401$, $\varepsilon_x = \frac{|E_x|}{x} = \frac{0.060401}{4} = 0.0151...$

From the relative error we can guess that one digit of our approximation is correct. This fits, because our approximation when rounded yields $f'(1) \approx 4.1$.

2. $h = \frac{2-0}{4} = \frac{1}{2}$.

Left rectangles: $\int\limits_0^2 \sqrt{x}\,dx \approx \frac{1}{2}\big[\sqrt{0} + \sqrt{0.5} + \sqrt{1} + \sqrt{1.5}\big]$.

Right rectangles: $\int\limits_0^2 \sqrt{x}\,dx \approx \frac{1}{2}\big[\sqrt{0.5} + \sqrt{1} + \sqrt{1.5} + \sqrt{2}\big]$.

Trapezoids: $\int\limits_0^2 \sqrt{x}\,dx \approx \frac{1}{2} \cdot \frac{1}{2}\big[\sqrt{0} + 2\sqrt{0.5} + 2\sqrt{1} + 2\sqrt{1.5} + \sqrt{2}\big]$.

Bonus answers: The rectangle methods are of order 1, the trapezoid method is of order 2.

A method is of order p if $|E_h| \le ch^p$, where E_h is the global error of an approximate solution obtained with (reasonably small) step h. Just to make it precise, the constant c depends on the problem that is being solved, but not on h.