DEN Homework # 4

Solve the problems and then bring your work to the lab in the fifth week of school.

1. Consider the initial value problem $y' = \dfrac{y}{2x}$, $y(1) = 1$.
Find a numerical approximation of this solution on $[1, 6]$ using the Euler method; namely, follow the following steps:
a) First, write general iterative formulas for x_i, y_i based on some (unknown to us) partition size $n \in \mathbb{N}$; this is a practice for the exam question "explain the Euler method" (you can also try to draw an explanatory picture).
b) Apply these formulas to the given problem and interval with partition size $n = 5$ and deduce dedicated iterative formulas.
c) Using these special formulas, calculate the first three points of the desired approximation (including the initial one, do the first two iterative steps).

2. We solved a certain initial value problem using the RK2 method (of order 2) with step size $h = 0.5$. We have a reason to think that the error is at most $E_{0.5} = 0.01$.
a) Estimate the error that appears when we apply this method with step size $h = 0.1$.
b) What step size would you recommend if we want the error to be below $E = 0.0025$?

By the way, the last two weeks we have been working on numerical analysis problems in our homeworks. Did you notice that all of them could be solved without the use of a calculator? You will not need one at the exam either (but it is not forbidden).