DEN Homework 04 – solution

1. a) This could do it: $h = \frac{b-a}{n}$.
(0) x_0, y_0 given by the initial condition;
(1) $x_{i+1} = x_i + h$, $y_{i+1} = y_i + f(x_i, y_i)h$ for $i = 0, \dots, n-1$.

b) For $\langle 1, 6 \rangle$ and $n = 5$ we get $h = 1$, hence
(0) $x_0 = 1$, $y_0 = 1$,
(1) $x_{i+1} = x_i + 1$, $y_{i+1} = y_i + \frac{y_i}{2x_i}$.

c) Points are $(1,1)$, $\big(2, \frac{3}{2}\big)$, $\big(3, \frac{15}{8}\big)$ or $(1,1)$, $(2, 1.5)$, $(3, 1.875)$ if you prefer.

2. For a method of order two, making the step size smaller five time should decrease the error approximately 5^2 times. The error estimate is therefore

$$E_{0.1} = \frac{1}{5^2} E_{0.5} = \frac{0.01}{25} = \frac{0.04}{100} = 0.0004.$$

b) Decreasing the step a-times means decreasing the error a^2 times for a method of order 2. How many times do we want to decrease the error? $\frac{E_{0.5}}{E} = \frac{0.01}{0.0025} = \frac{0.04}{0.01} = 4 = 2^2$.
If we want to decrease the error 2^2 times, so it is enough to decrease the step twice, that is, we recommend the step $h = \frac{1}{2} 0.5 = 0.25$. To be on the safe side we would probably take the step 0.2.
Other arguments are also possible. For instance, the known error tells us that $0.01 = c(0.5)^2$, hence $c = \frac{0.01}{0.25} = 0.04$. We want $0.0025 = ch^2 = 0.04h^2$, thus $h^2 = \frac{0.0025}{0.04} = \frac{1}{100}\frac{25}{4}$, and so

$$h = \sqrt{\frac{1}{100}\frac{25}{4}} = \frac{1}{10}\frac{5}{2} = 0.25.$$

Of course, if you have a calculator, you do not have to play like that and just pour some numbers into it.