

DEN Homework 06 – solution

1. 1) General solution:

a) Hom: $\lambda^2 - 5\lambda + 6 = 0 \implies \lambda = 2, 3$, $y_h(x) = a e^{2x} + b e^{3x}$, $x \in \mathbb{R}$.

b) Guessing: two types.

e^{3x} : base form $A e^{3x}$; $\lambda = 3$, a match of multiplicity $m = 1$, hence correction, $Ax e^{3x}$.

$12x - 4$: base form $Bx + C$; $\lambda = 0$, no match, hence no correction.

$y_p(x) = Ax e^{4x} + Bx + C$, substituting into the equation we get

$$A e^{3x} + 6Bx + [-5B + 6C] = e^{3x} + 12x - 4,$$

$A = 1$, $B = 2$, $C = 1$, a general solution is

$$y(x) = x e^{3x} + 2x + 1 + a e^{2x} + b e^{3x}, \quad x \in \mathbb{R}.$$

2) Init. conditions: $y(x) = x e^{3x} + 2x + 1 + e^{2x}$, $x \in \mathbb{R}$.

2. 1) General solution:

a) Hom: $\lambda^3 + 9\lambda = 0 \implies \lambda(\lambda^2 + 9) = 0 \implies \lambda = 0, \pm 3i$.

$\lambda = 0 \pm 3i$, $y_h(x) = a + b \cos(3x) + c \sin(3x)$, $x \in \mathbb{R}$.

b) Guessing: two types.

$8 \cos(x)$: base form $A \cos(x) + B \sin(x)$; $\lambda = i$, no match hence no correction.

-9 : base form C ; $\lambda = 0$, a match of multiplicity $m = 1$, hence correction, Cx .

$y_p(x) = A \cos(x) + B \sin(x) + Cx$, substituting into the equation we get

$$8B \cos(x) - 8A \sin(x) + 9C = 8 \cos(x) - 9,$$

$A = 0$, $B = 1$, $C = -1$, a general solution is

$$y(x) = \sin(x) - x + a + b \cos(3x) + c \sin(3x), \quad x \in \mathbb{R}.$$

2) Init. conditions: $y(x) = \sin(x) - x + 13$, $x \in \mathbb{R}$.