DEN Homework 11 – solution

1. a) $\left(\begin{array}{ccc|c} 3 & -2 & 1 & 7 \\ 1 & -1 & 0 & 2 \\ -4 & 2 & 0 & -6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 3 & -2 & 1 & 7 \\ -4 & 2 & 0 & -6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 4 \end{array}\right).$

Steps: $R1 \leftrightarrow R2$, $R2 - 3 \times R1 \mapsto R2$, $R3 + 4 \times R1 \mapsto R3$, $R3 + 2 \times R2 \mapsto R3$.

We get the system

$$\begin{aligned} x - y \quad &= 2 \\ y + z &= 1 \\ 2z &= 4. \end{aligned}$$

Back substitution: $z = \frac{1}{2} \cdot 4 = 2$, $y = 1 - z = -1$, $x = 2 + y = 1$.

b) The product of terms on the diagonal in the new matrix is 2. But we switched rows during elimination once, so the determinant of the original matrix of the system is $\det(A) = -2$.

This is the best way of finding determinants of huge matrices, as computational complexity $\frac{2}{3}n^3$ is definitely preferable to $n!$.

c) We apply the operations from the first elimination: $\begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 4 \\ -4 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$

We obtain the system

$$\begin{aligned} x - y \quad &= 1 \\ y + z &= 1 \\ 2z &= 2. \end{aligned}$$

Back substitution: $z = 1$, $y = 1 - z = 0$, $x = 1 + y = 1$. This checks out.

2. $\lambda = -1, 4$, $\varrho(A) = 4$. $\|A\|_\infty = \max(1 + 2, 3 + 2) = 5$, $\|A\|_1 = \max(1 + 3, 2 + 2) = 4$.

Note: These norms cannot be calculated using elimination, because row operations change norms!