DEN(E) Homework # 3 (surrogate)

1. Consider the function $f(x) = x^3$. Approximate its derivative at the point $a = 2$ using the forward difference with step $h = 0.01$.
You surely know the right value of this derivative. Find the absolute and relative error of your approximation.

2. Approximate the value of the integral $\int\limits_1^3 \frac{1}{x}\,dx$ using:
a) the method of left rectangles with $n = 4$;
b) the method of right rectangles with $n = 4$;
c) the trapezoid method with $n = 4$.
You do not have to conclude the calculations to get just a number, answers like $\frac{1}{7}[2^2 + 3^2]$ are enough (in fact we prefer them because then we can see what you did).

DEN(E) Homework # 4 (surrogate)

1. Consider the initial value problem $y' = \dfrac{3x^2}{2(y-1)}$, $y(1) = 2$.
Find a numerical approximation of this solution on $[1, 6]$ using the Euler method; namely, follow the following steps:
a) First, write general iterative formulas for x_i, y_i based on some (unknown to us) partition size $n \in \mathbb{N}$; this is a practice for the exam question "explain the Euler method" (you can also try to draw an explanatory picture).
b) Apply these formulas to the given problem and interval with partition size $n = 5$ and deduce dedicated iterative formulas.
c) Using these special formulas, calculate the first three points of the desired approximation (including the initial one, do the first two iterative steps).

2. We solved a certain initial value problem using the RK2 method (of order 2) with step size $h = 0.2$. We have a reason to think that the error is at most $E_{0.2} = 0.01$.
a) Estimate the error that appears when we apply this method with step size $h = 0.05$.
b) What step size would you recommend if we want the error to be below $E = 0.0025$?

DEN(E) Homework # 05 (surrogate)

1. Find the solution of the problem $y'' + 13y' = 0, \quad y(0) = 1,\ y'(0) = -13$.

2. Find a general solution of the equation $y''' - 3y'' + 4y' - 12y = 0$
and determine its typical asymptotic rate of growth at infinity.
Hint: $\lambda = 3$ could help.

DEN(E) Homework # 06 (surrogate)

1. Find the solution of the equation $y'' - 5y' + 6y = -e^{2x} + 6x - 11$ that satisfies the initial conditions $y(0) = 0,\ y'(0) = 4$.

2. Find the solution of the equation $y''' + 4y' = 4 - 6\sin(x)$ that satisfies the initial conditions $y(0) = 3,\ y'(0) = 1,\ y''(0) = -6$.

DEN(E) Homework # 07 (surrogate)

1. We are looking for the root of $f(x) = x^2 + x - 3$.

a) Apply the bisection method to this problem and the initial interval $[0, 4]$, show the first two iterative steps.
Comment on your steps, above all on your decision making, so that the examiner can see that you know what you are doing. (A novice should be able to understand how the bisection method works based on your comments.)

b) Apply the Newton method to this problem with the initial guess $x_0 = 0$. Find the first three approximations (that is, do two steps of iteration).

c) A customer asked for an approximation of a root with precision $\varepsilon = 0.5$. Is the number x_2 from part b) good enough?
(Using a calculator for this one is not considered cowardly.)

2. Apply the Newton method to the problem of finding a root of the function $f(x) = x^2 - x + 1$, with the initial guess $x_0 = 2$. First prepare and simplify a dedicated iterative formula and then find the first five approximations (that is, do four steps of iteration).
What do you think about this situation? Based on what you learned in the lecture, can you make a guess regarding the situation?

DEN(E) Homework # 08 (surrogate)

1. In the previous homework we looked for the root of the function $f(x) = x^2 + x - 3$ using the bisection method and the Newton method, there we had the initial guess $x_0 = 0$.

a) Find some transformation of the equation $x^2 + x - 3 = 0$ into a fixed-point problem. Write the corresponding iterative formula and show two iterative steps with initial value $x_0 = 0$ (that is, find x_1, x_2).
Then make a guess regarding the success of this iteration using $|\varphi'(0)|$.

b) Repeat the task a) for some other transformation of that equation into a fixed-point problem.

DEN(E) Homework # 09 (surrogate)

1. Find the solution of the initial value problem

$$\begin{aligned} y_1' &= 7y_1 - 6y_2 \qquad & y_1(0) &= 5 \\ y_2' &= 4y_1 - 3y_2, \qquad & y_2(0) &= 4. \end{aligned}$$

Use the matrix approach.
Is the trivial stationary solution $y_1(x) = 0$, $y_2(x) = 0$ of this system stable?

2. Find a general solution of the system

$$\begin{aligned} y_1' &= 9y_2 \\ y_2' &= -y_1, \end{aligned}$$

Use the matrix approach.
Is the trivial stationary solution $y_1(x) = 0$, $y_2(x) = 0$ of this system stable?
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DEN(E) Homework # 11 (surrogate)

1. Consider the following system.

$$\begin{aligned} 4x + 6y + 8z &= 14 \\ -3x - 5y - 4z &= -6 \\ x + 2y + z &= 1. \end{aligned}$$

a) Solve this system using elimination in the way it is used in numerical mathematics, that is, first GEM and then the back substitution.
While doing the elimination, note down the steps that you applied. The form of those notes is up to you, but they should allow you to recreate your elimination.

b) Determine the determinant of the matrix of the system, use the row-echelon form for it.

c) We are given a new system

$$\begin{aligned} 4x + 6y + 8z &= 12 \\ -3x - 5y - 4z &= -7 \\ x + 2y + z &= 1, \end{aligned}$$

that has the same left-hand sides as the previous one (what a remarkable coincidence). Apply the row operations that you noted down in part a) just to the vector of the right-hand sides of the new systen. Connect the resulting vector with the reduced upper-triangular matrix from part a) and use the backward elimination to find the solution.
Check by substituting into the given system that you indeed found the right solution.

2. Consider the matrix $A = \begin{pmatrix} 7 & -2 \\ 4 & 1 \end{pmatrix}$.
Find its eigenvalues and the spectral radius.
Calculate its row norm and column norm.

DEN(E) Homework # 12 (surrogate)

1. We are given the following system:

$$\begin{aligned} x + y + 2z &= -1 \\ 4x + y + z &= 7 \\ x + 3y - z &= 7. \end{aligned}$$

By the way, its solution is $x = 2$, $y = 1$, $z = -2$.

a) Rewrite this system into a form suitable for the Jacobi iterative method. Given the initial vector $x_0 = 0$, $y_0 = 3$, $z_0 = 0$, calculate the next three iterations of the Jacobi iteration. Do you get the feeling that it would converge to the solution?

b) Apply the Gauss-Seidel iteration to this system. Namely, calculate the first two iterations given the initial vector $x_0 = 0$, $y_0 = 3$, $z_0 = 0$. Add a remark outlining the key difference compared to the Jacobi iteration at a suitable place.

c) Reorder the system so that the Jacobi and Gauss-Seidel iterations have a high chance of success. Explain what shape of a system you are trying to achieve.

Apply the Jacobi iteration to the reordered system and calculate the next three iterations based on the initial vector $x_0 = y_0 = z_0 = 0$. Does it look hopeful?

e) Apply the Gauss-Seideliteration to the reordered system and calculate the next two iterations based on the initial vector $x_0 = 0$, $y_0 = 3$, $z_0 = 0$. Does it look hopeful?

e) Apply the Gauss-Seideliteration to the reordered system and calculate the next two iterations based on the initial vector $x_0 = 0$, $y_0 = 3$, $z_0 = 0$. Does it look hopeful?