DEN: PDE

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fy = f.$$

- **elliptic** equation: $B^2 - 4AC < 0$

  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f$ $\qquad \Delta u = f$

- **parabolic** equation: $B^2 - 4AC = 0$

  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f$ $\qquad \frac{\partial u}{\partial t} = \Delta u + f$

• **hyperbolic** equation: $B^2 - 4AC > 0$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f \qquad \frac{\partial^2 u}{\partial t^2} = \Delta u + f$$

Fact.

$$\frac{\partial u}{\partial t}(t,x,y) = \frac{u(t+h,x,y) - u(t,x,y)}{h} + O(h),$$

$$\frac{\partial u}{\partial t}(t,x,y) = \frac{u(t+h,x,y) - u(t-h,x,y)}{2h} + O(h^2),$$

$$\frac{\partial^2 u}{\partial x^2}(t,x,y) = \frac{u(t,x+h,y) + u(t,x-h,y) - 2u(t,x,y)}{h^2} + O(h^2),$$

$$\frac{\partial^2 u}{\partial x \partial y}(t,x,y) =$$

$$= \frac{u(t,x+h,y+h) - u(t,x+h,y-h) - u(t,x-h,y+h) + u(t,x-h,y-h)}{4h^2}$$

$$+ O(h^2).$$

$$\Delta u \approx \frac{u(t,x+h,y) + u(t,x-h,y) + u(t,x,y+h) + u(t,x,y-h) - 4u(t,x,y)}{h^2}.$$